Maths Chapter 11 Section Formula Ex 11

Question 1.
Find the co-ordinates of the mid-point of the line segments joining the following pairs of points:
(i) (2, -3), (-6, 7)
(ii) (5, -11), (4, 3)
(iii) (a + 3, 5b), (2a – 1, 3b + 4)
Solution:

(i) Co-ordinates of the mid-point of (2, -3), (-6, 7)

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \text{ or}$$

$$\left(\frac{2-6}{2}, \frac{-3+7}{2}\right) \text{ or } \left(\frac{-4}{2}, \frac{4}{2}\right) \text{ or } (-2,2)$$

(ii) Mid-point of (5, – 11) and (4, 3)

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{or } \left(\frac{5+4}{2}, \frac{-11+3}{2}\right)$$

$$\text{or } \left(\frac{9}{2}, \frac{-8}{2}\right) \text{ or } \left(\frac{9}{2}, -4\right)$$

(iii) Mid-point of (a + 3, 5 b) and (2 a – 1, 3 b + 4)

$$= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$\text{or } \left(\frac{a+3+2a-1}{2}, \frac{5b+3b+4}{2}\right)$$

$$\text{or } \left(\frac{3a+2}{2}, \frac{8b+4}{2}\right)$$

$$\text{or } \left(\frac{3a+2}{2}, (4b+2)\right)$$

Question 2.
The co-ordinates of two points A and B are (-3, 3) and (12, -7) respectively. P is a point on the line segment AB such that AP : PB = 2 : 3. Find the co-ordinates of P.
Solution:

Points are A (-3, 3), B (12, -7)

Let P (x_1, y_1) be the point which divides AB in the ratio of $m_1 : m_2$ i.e. 2 : 3

then co-ordinates of P will be

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times12 + 3\times(-3)}{2+3}$$

$$= \frac{24-9}{5} = \frac{15}{5} = 3$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times(-7) + 3(3)}{2+3}$$

$$= \frac{-14+9}{5} = \frac{-5}{5} = -1$$

$\therefore$ Co-ordinates of P are (3, – 1)

Question 3.
P divides the distance between A (-2, 1) and B (1, 4) in the ratio of 2 : 1. Calculate the co-ordinates of the point P.

Solution:

Points are A (-2, 1) and B (1, 4) and

Let P (x, y) divides AB in the ratio of $m_1 : m_2$ i.e. 2 : 1

Co-ordinates of P will be

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times1 + 1\times(-2)}{2+1}$$

$$= \frac{2-2}{3} = \frac{0}{3} = 0$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{2\times4 + 1\times1}{2+1} = \frac{8+1}{3} = \frac{9}{3} = 3$$

$\therefore$ Co-ordinates of point P are (0, 3).

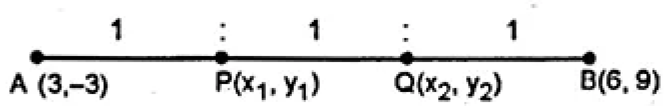
Question 4.

(i) Find the co-ordinates of the points of trisection of the line segment joining the point (3, -3) and (6, 9).

(ii) The line segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, – 2) and (53,q) respectively, find the values of p and q.

Solution:

(i) Let P (x_1, y_1) and Q (x_2, y_2) be the points which trisect the line segment joining the points A (3, -3) and B (6, 9)



$\because$ P (x_1, y_1) divides AB in the ratio of 1 : 2

$$\therefore\ x_1 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{1\times6+2\times3}{1+2} = \frac{6+6}{3} = \frac{12}{3} = 4$$

$$y_1 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1\times9+2\times(-3)}{1+2}$$

$$= \frac{9-6}{3} = \frac{3}{3} = 1$$

$\therefore$ Co-ordinates of P are (4, 1)

Again $\because$ Q (x_2, y_2) divides the line segment AB in the ratio of 2 : 1

$$\therefore\ x_2 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

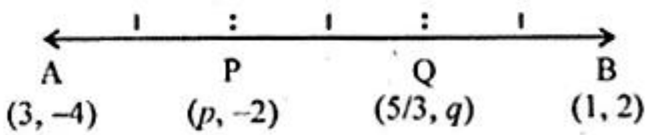
$$= \frac{2\times6+1\times3}{2+1} = \frac{12+3}{3} = \frac{15}{3} = 5$$

$$y_2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times9+1(-3)}{2+1}$$

$$= \frac{18-3}{3} = \frac{15}{3} = 5$$

$\therefore$ Co-ordinates of Q are (5, 5)

(*ii*) Points P and Q trisect the line AB.



In other words, P divides it in the ratio 1 : 2
and Q divides it in the ratio 2 : 1

$$\therefore\ p = \frac{mx_2 + nx_1}{m+n} = \frac{1\times1+2\times3}{1+2} = \frac{1+6}{3} = \frac{7}{3}$$

$$q = \frac{my_2 + ny_1}{m+n} = \frac{2\times2+1\times(-4)}{2+1} = \frac{4-4}{2} = 0$$

$$\therefore\ p = \frac{7}{3},\ q = 0$$

Question 5.
(i) The line segment joining the points A (3, 2) and B (5, 1) is divided at the point P in the ratio 1 : 2 and it lies on the line 3x – 18y + k = 0. Find the value of k.
(ii) A point P divides the line segment joining the points A (3, -5) and B (-4, 8) such that $\frac{AP}{PB}=\frac{k}{1}$ If P lies on the line x + y = 0, then find the value of k.
Solution:

(i) The point P (x, y) divides the line segment joining the points A (3, 2) and B (5, 1) in the ratio 1 : 2

$$\therefore\ x = \frac{mx_2 + nx_1}{m+n} = \frac{1\times5+2\times3}{1+2}$$

$$= \frac{5+6}{3} = \frac{11}{3}$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{1\times1+2\times2}{1+2}$$

$$= \frac{1+4}{3} = \frac{5}{3}$$

$\because$ P lies on the line $3x - 18y + k = 0$

$\therefore$ It will satisfy it.

$$3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$

$$11 - 30 + k = 0 \Rightarrow -19 + k = 0$$

$$k = 19$$

(*ii*) A point P divides the line segment joining the points A (3, –5), B (–4, 8) such that $\frac{AP}{BP} = \frac{k}{1}$

$\therefore$ Ratio = AP : PB = k : 1

Let co-ordinates of P be (x, y), then

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{k \times (-4) + 1 \times 3}{k+1}$$

$$x = \frac{-4k+3}{k+1}.$$

$$\text{and } y = \frac{8k-5}{k+1} \qquad \left\{\because y = \frac{my_2 + ny_1}{m+n}\right\}$$

$$= \frac{8k-5}{k+1}$$

$\because$ This point lies on the line $x + y = 0$

$$\therefore \frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$$

$$\Rightarrow -4k + 3 + 8k - 5 = 0$$

$$\Rightarrow 4k - 2 = 0 \Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Question 6.
Find the coordinates of the point which is three-fourths of the way from A (3, 1) to B (-2, 5).
Solution:

Let P be the required point, then

$$\frac{AP}{AB} = \frac{3}{4}$$

and co-ordinates of A are (3, 1) and of B are (–2, 5)

$$\therefore \quad \frac{AP}{AB} = \frac{3}{4} = \frac{AP}{AP + PB} = \frac{3}{4}$$

$$\Rightarrow \quad 4AP = 3AP + 3PB$$

$$\Rightarrow \quad 4AP - 3AP = 3PB$$

$$AP = 3PB$$

$$\frac{AP}{PB} = \frac{3}{1}$$

$$\therefore \quad m_1 = 3, m_2 = 1$$

Let co-ordinates of P be (x, y)

$$\therefore \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{3 \times (-2) + 1 \times (3)}{3 + 1}$$

$$= \frac{-6 + 3}{4} = \frac{-3}{4}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{3 \times 5 + 1 \times 1}{3 + 1}$$

$$= \frac{15 + 1}{4} = \frac{16}{4} = 4$$

$\therefore$ Co-ordinates of P will be $\left(\frac{-3}{4}, 4\right)$

Question 7.
Point P (3, -5) is reflected in P’ in the x-axis. Also, P on reflection in the y-axis is mapped as P”.

(i) Find the co-ordinates of P' and P".
(ii) Compute the distance P' P".
(iii) Find the middle point of the line segment P' P".
(iv) On which co-ordinate axis does the middle point of the line segment P P" lie?
Solution:

(i) Co-ordinates of P', the image of P (3, -5)
when reflected in x-axis will be (3, 5)
and co-ordinates of P", the image of P (3, -5)
when reflected in y-axis will be (-3, -5)

when reflected in y-axis will be $(-3, -5)$

(ii) Length of $P'P'' = \sqrt{(-3-3)^2 - (-5-5)^2}$

$= \sqrt{(-6)^2 + (-10)^2} = \sqrt{36 + 100}$

$= \sqrt{136} = \sqrt{4 \times 34} = 2\sqrt{34}$ units

(iii) Let co-ordinates of middle point M be (x, y)

$$\therefore\ x = \frac{x_1 + x_2}{2} = \frac{3-3}{2} = \frac{0}{2} = 0$$

$$y = \frac{y_1 + y_2}{2} = \frac{-5+5}{2} = \frac{0}{2} = 0$$

$\therefore$ middle point is (0, 0)

(iv) Middle point of PP″ be N (x_1, y_1)

$$\therefore\ x_1 = \frac{3-3}{2} = \frac{0}{2} = 0$$

$$x_2 = \frac{-5-5}{2} = \frac{-10}{2} = -5$$

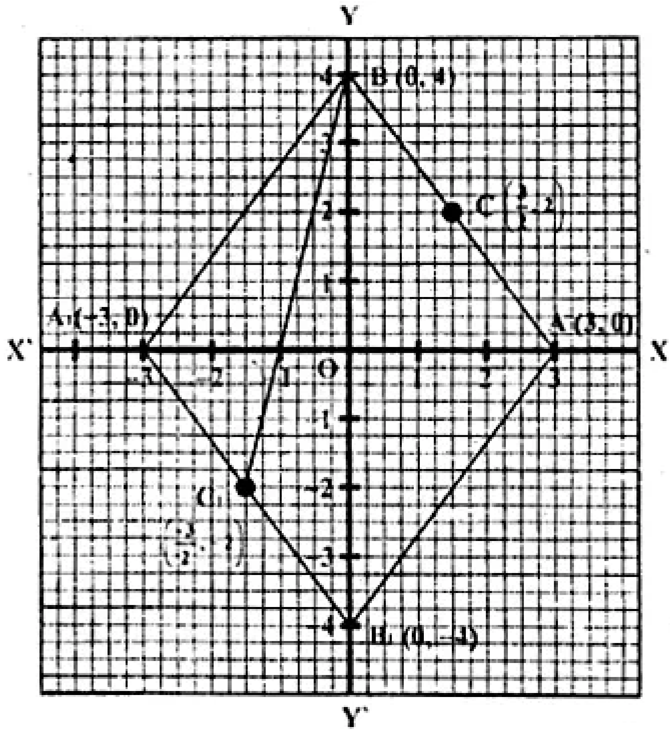
$\therefore$ Co-ordinates of middle point of PP″ are $(0, -5)$

As $x = 0$, this point lies on y-axis

Question 8.
Use graph paper for this question. Take 1 cm = 1 unit on both axes. Plot the points A(3, 0) and B(0, 4).
(i) Write down the co-ordinates of A1, the reflection of A in the y-axis.

(ii) Write down the co-ordinates of B1, the reflection of B in the x-axis.
(iii) Assign the special name to the quadrilateral ABA1B1.
(iv) If C is the midpoint is AB. Write down the co-ordinates of the point C1, the reflection of C in the origin.
(v) Assign the special name to quadrilateral ABC1B1.
Solution:

Two points A (3, 0) and B (0,4) have been plotted on the graph.



(i)∵ A1 is the reflection of A (3, 0) in the v-axis Its co-ordinates will be (-3, 0)

(ii)∵ B1 is the reflection of B (0, 4) in the .x-axis co-ordinates of B, will be (0, -4)

(iii) The so formed figure ABA1B1 is a rhombus.

(iv) C is the mid point of AB co-ordinates of C" will be $\frac{AP}{AB} = \frac{3}{4}$

∵ C, is the reflection of C in the origin

co-ordinates of C, will be $\left(\frac{-3}{2}, -2\right)$

(v) The name of quadrilateral ABC1B1 is a trapezium because AB is parallel to B1C1.

Question 9.
The line segment joining A (-3, 1) and B (5, -4) is a diameter of a circle whose centre is C. find the co-ordinates of the point C. (1990)

Solution:

∵ C is the centre of the circle and AB is the diameter

C is the midpoint of AB.

Let co-ordinates of C (x, y)

$$\therefore \quad x = \frac{-3+5}{2}, \; x = \frac{1-4}{2}$$

$$\Rightarrow \quad x = \frac{2}{2}, \; y = \frac{-3}{2}$$

$$\Rightarrow \quad x = 1, \; y = \frac{-3}{2}$$

$\therefore$ Co-ordinates of C are $\left(1, \frac{-3}{2}\right)$

Question 10.

The mid-point of the line segment joining the points (3m, 6) and (-4, 3n) is (1, 2m – 1). Find the values of m and n.

Solution:

Let the mid-point of the line segment joining two points

A(3m, 6) and (-4, 3n) is P(1, 2m − 1)

$$\therefore \quad 1 = \frac{x_1 + x_2}{2} = \frac{3m-4}{2}$$

$$\Rightarrow \quad 3m - 4 = 2 \qquad \Rightarrow 3m = 2 + 4 = 6$$

$$\Rightarrow \quad m = \frac{6}{3} = 2$$

$$\text{and } 2m - 1 = \frac{6+3n}{2} \qquad \Rightarrow 4m - 2 = 6 + 3n$$

$$\Rightarrow \quad 4 \times 2 - 2 = 6 + 3n = 8 - 2 = 6 + 3n$$

$$\Rightarrow \quad 3n = 8 - 2 - 6 = 0 \qquad \Rightarrow n = 0$$

Hence $m = 2, n = 0$

Question 11.

The co-ordinates of the mid-point of the line segment PQ are (1, -2). The co-ordinates of P are (-3, 2). Find the co-ordinates of Q.(1992)

Solution:
Let the co-ordinates of Q be (x, y)
co-ordinates of P are (-3, 2) and mid-point of PQ are (1, -2) then

$$1 = \frac{-3+x}{2} \Rightarrow -3+x=2 \quad \Rightarrow \quad x=2+3=5$$

$$\text{and } -2 = \frac{2+y}{2} \Rightarrow 2+y=-4 \Rightarrow \; y=-4-2=-6$$

$\therefore x=5, \; y=-6$

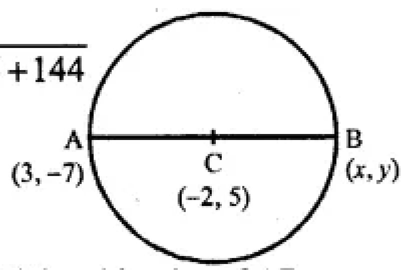
Hence co-ordinates of Q are (5, – 6)

Question 12.
AB is a diameter of a circle with centre C (-2, 5). If point A is (3, -7). Find:
(i) the length of radius AC.
(ii) the coordinates of B.
Solution:

$$AC = \sqrt{(3+2)^2 + (-7-5)^2}$$

$$= \sqrt{5^2+12^2} = \sqrt{25+144}$$

$$= \sqrt{169} = 13 \text{ units}$$



$\because$ AB is diameter and C is mid point of AB

Let co-ordinate of B are (x, y)

$$\therefore \frac{3+x}{2} = -2 \text{ and } \frac{y-7}{2} = 5$$

$3 + x = -4$ and $y - 7 = 10$

$x = -4 - 3$ and $y = 10 + 7$

$x = -7$ and $y = 17$

$\therefore$ B is (–7, 17)

Question 13.
Find the reflection (image) of the point (5, -3) in the point (-1, 3).

Solution:

Let the co-ordinates of the images of the point A (5, -3) be

A1 (x, y) in the point (-1, 3) then

the point (-1, 3) will be the midpoint of AA1.

$$\therefore -1 = \frac{5+x}{2} \Rightarrow 5+x=-2 \quad \Rightarrow \quad x=-2-5=-7$$

$$\text{and } 3 = \frac{-3+y}{2} \Rightarrow -3+y=6 \Rightarrow \quad y=6+3=9$$

$\therefore$ Co-ordinates of the image A, will be $(-7, 9)$.

Question 14.

The line segment joining A $(-1, \frac{5}{3})$ the points B (a, 5) is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects y-axis. Calculate

(i) the value of a

(ii) the co-ordinates of P. (1994)

Solution:

Let P (x, y) divides the line segment joining

the points $\left(-1, \frac{5}{3}\right)$, B(a, 5) in the ratio 1 : 3

$$\therefore x = \frac{1\times a + 3\times(-1)}{1+3} = \frac{a-3}{4}$$

$$y = \frac{1\times a + 3\times(-1)}{1+3} = \frac{a-3}{4} = \frac{5+5}{4} = \frac{10}{4} = \frac{5}{2}$$

(i) $\because$ AB intersects y-axis at P

$$\therefore x=0 \Rightarrow \frac{a-3}{4} = 0 \Rightarrow a-3=0$$

$$\therefore a=3$$

(ii) $\therefore$ Co-ordinates of P are $\left(0, \frac{5}{2}\right)$

Question 15.

The point P (-4, 1) divides the line segment joining the points A (2, -2) and B in the ratio of 3 : 5. Find the point B.

Solution:

Let the co-ordinates of B be (x, y)

Co-ordinates of A (2, -2) and point P (-4, 1)

divides AB in the ratio of 3 : 5

$$\therefore -4 = \frac{3 \times x + 5 \times (2)}{3+5} = \frac{3x+10}{8}$$

and $3x + 10 = -32 \Rightarrow 3x = -32 - 10 = -42$

$$\therefore \quad x = \frac{-42}{3} = -14$$

$$1 = \frac{3 \times y + 5 \times (-2)}{3+5} \Rightarrow 1 = \frac{3y-10}{8}$$

$\Rightarrow \; 3y - 10 = 8 \Rightarrow 3y = 8 + 10 = 18$

$$\therefore \quad y = \frac{18}{3} = 6$$

$\therefore$ Co-ordinates of B $= (-14, 6)$

Question 16.

(i) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7 ,6) ?

(ii) In what ratio does the point (-4, b) divide the line segment joining the points P (2, -2), Q (-14, 6) ? Hence find the value of b.

Solution:

(i) Let the ratio be $m_1 : m_2$ that the point (5, 4) divides the line segment joining the points (2, 1), (7, 6).

$$5 = \frac{m_1 \times 7 + m_2 \times 2}{m_1 + m_2}$$

$$\Rightarrow 5m_1 + 5m_2 = 7m_1 + 2m_2$$
$$\Rightarrow 5m_2 - 2m_2 = 7m_1 - 5m_1 \Rightarrow 3m_2 = 2m_1$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{3}{2} \quad \Rightarrow m_1 : m_2 = 3:2$$

(ii) The point (– 4, b) divides the line segment joining the points P (2, – 2) and Q (– 14, 6) in the ratio $m_1 : m_2$.

$$\therefore -4 = \frac{m_1(-14) + m_2 \times 2}{m_1 + m_2}$$

$$\Rightarrow -4m_1 - 4m_2 = -14m_1 + 2m_2$$
$$\Rightarrow -4m_1 + 14m_1 = 2m_2 + 4m_2 \Rightarrow 10m_1 = 6m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{6}{10} = \frac{3}{5} \Rightarrow m_1 : m_2 = 3:5$$

Again

$$b = \frac{m_1 \times 6 + m_2 \times (-2)}{m_1 + m_2} = \frac{6m_1 - 2m_2}{m_1 + m_2}$$

$$\Rightarrow b = \frac{6 \times 3 - 2 \times 5}{3+5} = \frac{18-10}{8} = \frac{8}{8} = 1$$

$$\therefore b = 1$$

Question 17.

The line segment joining A (2, 3) and B (6, -5) is intercepted by the x-axis at the point K. Write the ordinate of the point k. Hence, find the ratio in which K divides AB. Also, find the coordinates of the point K.

Solution:|

Let the co-ordinates of K be (x, 0) as it intersects x-axis.

Let point K divides the line segment joining the points A (2, 3) and B (6, -5) in the ratio $m_1 : m_2$.

$$\therefore 0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \Rightarrow 0 = \frac{m_1 \times (-5) + m_2 \times 3}{m_1 + m_2}$$

$$\Rightarrow -5\, m_1 + 3\, m_2 = 0 \Rightarrow -5\, m_1 = -3\, m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{3}{5} \Rightarrow m_1 : m_2 = 3 : 5$$

Now,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{3 \times 6 + 5 \times 2}{3 + 5} = \frac{18 + 10}{8} = \frac{28}{8} = \frac{7}{2}$$

Co-ordinate of K are $\left(\frac{7}{2}, 0\right)$

Question 18.

If A (-4, 3) and B (8, -6)

(i) find the length of AB.

(ii) in what ratio is the line joining AB, divided by the x-axis? (2008)

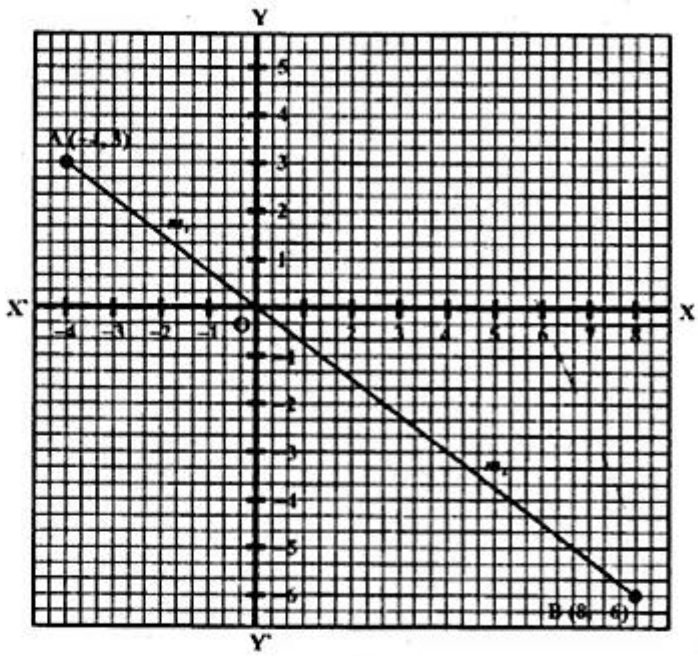
Solution:

Given A (-4, 3), B (8, -6)

$$\therefore \text{ Length of AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{[8-(-4)]^2+(-6-3)^2}=\sqrt{(8+4)^2+(-6-3)^2}$$

$$=\sqrt{(12)^2+(-9)^2}=\sqrt{144+81}=\sqrt{225}=15$$



By joining AB, we see
that O (0, 0) lies on AB
Let O divides AB in the ratio $m_1 : m_2$

$$\therefore x=\frac{m_1x_2+m_2x_1}{m_1+m_2} \Rightarrow 0=\frac{m_1\times 8+m_2(-4)}{m_1+m_2}$$

$$\Rightarrow 8m_1-4m_2=0 \quad \Rightarrow 8m_1=4m_2 \Rightarrow \quad \frac{m_1}{m_2}=\frac{4}{8}=\frac{1}{2}$$

$\therefore\ m_1 : m_2 = 1 : 2$

$\therefore$ O, divides AB in the ratio 1 : 2

Question 19.
(i) Calculate the ratio in which the line segment joining (3, 4) and(-2, 1) is divided by the y-axis.
(ii) In what ratio does the line x – y – 2 = 0 divide the line segment joining the

points (3, -1) and (8, 9)?
Also, find the coordinates of the point of division.
Solution:

(i) Let the point P divides the line segment joining the points A (3, 4) and B (-2, 3) in the ratio of $m_1 : m_2$ and let the co-ordinates of P be (0, y) as it intersects the y-axis

$$\therefore 0 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{m_1(-2) + m_2 \times 3}{m_1 + m_2} \Rightarrow 0 = -2m_1 + 3m_2$$

$$\Rightarrow 2m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{2} \Rightarrow m_1 : m_2 = 3 : 2 \text{ Ans.}$$

(*ii*) Let the points be A (3, –1) and B (8, 9) and let line $x - y - 2 = 0$ divides the line segment joining the points A and B in the ratio $m_1 : m_2$ at point P (x, y) then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{m_1 \times 8 + m_2 \times 3}{m_1 + m_2}$$

and y ·

$$= \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{m_1 \times 9 + m_2(-1)}{m_1 + m_2} = \frac{9m_1 - m_2}{m_1 + m_1}$$

$\because$ The point P (x, y) lies on the line $x - y - 2 = 0$

$$\therefore \frac{8m_1 + 3m_2}{m_1 + m_2} - \frac{9m_1 - m_2}{m_1 + m_2} - 2 = 0$$

$$\Rightarrow 8m_1 + 3m_2 - 9m_1 + m_2 - 2m_1 - 2m_2 = 0$$

$$\Rightarrow -3m_1 + 2m_2 = 0 \Rightarrow 3m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$$

(*i*) $\because$ Ratio $= m_1 : m_2 = 2 : 3$

$$\therefore x = \frac{2 \times 8 + 3 \times 3}{2 + 3} = \frac{16 + 9}{5} = \frac{25}{5} = 5$$

$$\text{and } y = \frac{2 \times 9 + 3 \times (-1)}{2 + 3} = \frac{18 - 3}{5} = \frac{15}{5} = 3$$

(*ii*) $\therefore$ Co-ordinates of point P are (5, 3)

Question 20.
Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:
(i) the ratio in which AB is divided by the y-axis.
(ii) find the coordinates of the point of intersection.
(iii)the length of AB.
Solution:

(i) Let the y-axis divide AB in the ratio m : 1. So,

$$0=\frac{m\times 8-4\times 1}{m+1}\Rightarrow 8m-4=0\Rightarrow m=\frac{4}{8}\Rightarrow m=\frac{1}{2}$$

So, required ratio $=\frac{1}{2}:1$ or $1:2$

(ii) Also, $y=\frac{1\times(-3)+2\times 6}{1+2}=\frac{9}{3}=3$

So, coordinates of the point of intersection are (0, 3)

(iii) $AB=\sqrt{(8+4)^2+(-3-6)^2}$

$=\sqrt{144+81}=\sqrt{225}=15$ units

Question 21.
(i) Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17, 10) in ratio 1 : 2.
(ii)Calculate the distance OP where O is the origin.
(iii)In what ratio does the y-axis divide the line AB?

Solution:

(i) Let co-ordinate of P be (x, y) which divides the line segment joining the points A (-4, 1) and B(17, 10) in the ratio of 1 : 2.

$$\therefore x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$= \frac{1\times17 + 2\times(-4)}{1+2} = \frac{17-8}{3} = \frac{9}{3} = 3$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times10 + 2\times1}{1+2} = \frac{10+2}{3} = \frac{12}{3} = 4$$

$\therefore$ Co-ordinates of P are (3, 4)

(*ii*) Distance of OP where O is the origin *i.e.* co-ordinates are (0, 0)

$$\therefore \text{Distance} = \sqrt{(3-0)^2 + (4-0)^2}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

(*iii*) Let y-axis divides AB in the ratio of $m_1 : m_2$ at P and let co-ordinates of P be $(0, y)$

$$0 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \Rightarrow 0 = \frac{m_1 \times 17 + m_2 \times (-4)}{m_1 + m_2}$$

$$\Rightarrow 17m_1 - 4m_2 = 0 \Rightarrow 17m_1 = 4m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{4}{17} \Rightarrow m_1 : m_2 = 4 : 17$$

Question 22.

Calculate the length of the median through the vertex A of the triangle ABC with vertices A (7, -3), B (5, 3) and C (3, -1)

Solution:

Let D (x, y) be the median of ΔABC through A to BC.

∴ D will be the midpoint of BC

∴ Co-ordinates of D will be,

$$x = \frac{5+3}{2} = \frac{8}{2} = 4 \text{ and } y = \frac{3-1}{2} = \frac{2}{2} = 1$$

Co-ordinates of D are (4, 1)

$$\therefore \text{Length of DA} = \sqrt{(7-4)^2 + (-3-1)^2}$$

$$= \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units.}$$

Question 23.

Three consecutive vertices of a parallelogram ABCD are A (1, 2), B (1, 0) and C (4, 0). Find the fourth vertex D.

Solution:

Let O in the mid-point of AC the diagonal of ABCD

∴ Co-ordinates of O will be

$$\left(\frac{1+4}{2}, \frac{2+0}{2}\right) \text{ or } \left(\frac{5}{2}, 1\right)$$

∵ OA also the mid point of second diagonal BD and let co-ordinates of D be (x, y)

$$\therefore \frac{5}{2} = \frac{1+x}{2} \Rightarrow 10 = 2 + 2x \Rightarrow 2x = 10 - 2 = 8$$

$$\therefore x = \frac{8}{2} = 4 \qquad \text{and } 1 = \frac{0+y}{2} \Rightarrow y = 2$$

∴ Co-ordinates of D are (4, 2)

Question 24.

If the points A (-2, -1), B (1, 0), C (p, 3) and D (1, q) from a parallelogram ABCD, find the values of p and q.

Solution:

A (-2, -1), B (1, 0), C (p, 3) and D (1, q)

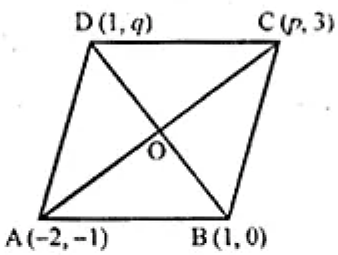
are the vertices of a parallelogram ABCD

∴ Diagonal AC and BD bisect each other at O

O is the midpoint of AC as well as BD

Let co-ordinates of O be (x, y)

When O is mid-point of AC, then



$\therefore\ x = \frac{p-2}{2}, y = \frac{3-1}{2} = \frac{2}{2} = 1$

Again when O is the mid-point of BD

Then $x = \frac{1+1}{2} = \frac{2}{2} = 1$ and $y = \frac{0+q}{2} = \frac{q}{2}$

Now comparing, we get

$\frac{p-2}{2} = 1 \Rightarrow p - 2 = 2 \Rightarrow p = 2 + 2 = 4$

$\therefore\ p = 4$ and $\frac{q}{2} = 1 \Rightarrow q = 2$

Hence $p = 4, q = 2$

Question 25.

If two vertices of a parallelogram are (3, 2) (-1, 0) and its diagonals meet at (2, -5), find the other two vertices of the parallelogram.

Solution:

Two vertices of a ||gm ABCD are A (3, 2), B (-1, 0)

and point of intersection of its diagonals is P (2, -5)

P is mid-point of AC and BD.

Let co-ordinates of C be (x, y), then

$$2 = \frac{x+3}{2} \Rightarrow x + 3 = 4 \Rightarrow x = 4 - 3 = 1$$

$$\text{and } -5 = \frac{y+2}{2} \Rightarrow y + 2 = -10$$

$$\Rightarrow y = -10 - 2 = -12$$

$\therefore$ Co-ordinates of C are (1, −12)

Similarly we shall find the co-ordinates of D also

$$2 = \frac{x-1}{2} \Rightarrow x - 1 = 4 \Rightarrow x = 4 + 1 = 5$$

$$-5 = \frac{y+0}{2} \Rightarrow -10 = y$$

$\therefore$ Co-ordinates of D are (5, −10)

Question 26.

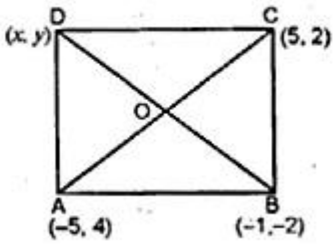
Prove that the points A (-5, 4), B (-1, -2) and C (5, 2) are the vertices of an isosceles right-angled triangle. Find the coordinates of D so that ABCD is a square.

Solution:

Points A (-5, 4), B (-1, -2) and C (5, 2) are given.

If these are vertices of an isosceles triangle ABC then

AB = BC.



$$AB = \sqrt{[-1-(-5)]^2 + (-2-4)^2}$$

$$= \sqrt{(-1+5)^2 + (-6)^2} = \sqrt{(4)^2 + (-6)^2}$$

$$= \sqrt{16+36} = \sqrt{52}$$

$$BC = \sqrt{[5-(-1)]^2 + [2-(-2)^2]}$$

$$= \sqrt{(5+1)^2 + (2+2)^2}$$

$$= \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} \quad \because AB = BC$$

$\therefore$ Δ ABC is an isosceles triangle.

$$AC = \sqrt{(-5-5)^2 + (4-2)^2}$$

$$= \sqrt{(-10)^2 + (2)^2} = \sqrt{100+4} = \sqrt{104}$$

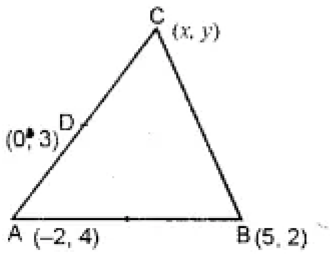
Now $AC^2 = AB^2 + BC^2$

Question 27.

Find the third vertex of a triangle if its two vertices are (-1, 4) and (5, 2) and the midpoint of one side is (0, 3).

Solution:

Let A (-1, 4) and B (5, 2) be the two points and let D (0, 3) be its the midpoint of AC and co-ordinates of C be (x, y).



$\therefore\ 0 = \frac{x-1}{2} \Rightarrow x - 1 = 0 \Rightarrow x = 1$

$3 = \frac{y+4}{2} \Rightarrow y + 4 = 6 \Rightarrow y = 6 - 4 = 2$

$\therefore$ Co-ordinates of will be (1, 2)

If we take mid-point D (0, 3) of BC, then

$0 = \frac{5+x}{2} \Rightarrow x + 5 = 0 \Rightarrow x = -5$

and $3 = \frac{2+y}{2} \Rightarrow 2 + y = 6 \Rightarrow y = 6 - 2 = 4$

$\therefore$ Co-ordination of C will be (–5, 4)

Hence co-ordinates of C, third vertx will be (1, 2) or (– 5, 4)

Question 28.

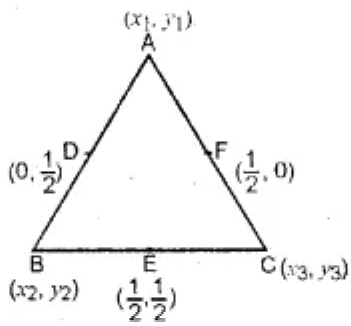
Find the coordinates of the vertices of the triangle the middle points of whose sides are (0,12),(12,12)and(12,0)

Solution:

Let ABC be a Δ in which $D\left(0,\frac{1}{2}\right), E\left(\frac{1}{2},\frac{1}{2}\right) and F\left(\frac{1}{2},0\right)$,

the mid-points of sides AB, BC and CA respectively.

Let co-ordinates of A be (x_1, y_1), B (x_2, y_2), C (x_3, y_3)



$$0\frac{x_1+x_2}{2} \Rightarrow x_1+x_2=0 \qquad \ldots(i)$$

$$\frac{1}{2}=\frac{y_1+y_2}{2} \Rightarrow y_1+y_2=1 \qquad \ldots(ii)$$

$$\text{Again } \frac{1}{2}=\frac{x_2+x_3}{2}=x_2+x_3=1 \qquad \ldots(iii)$$

$$\text{and } \frac{1}{2}=\frac{y_2+y_3}{2} \Rightarrow y_2+y_3=1 \qquad \ldots(iv)$$

$$\text{and } \frac{1}{2}=\frac{x_3+x_1}{2} \Rightarrow x_3+x_1=1 \qquad \ldots(v)$$

$$0=\frac{y_3+y_1}{2} \Rightarrow y_3+y_1=0 \qquad \ldots(vi)$$

Adding (i), (iii) and (v)

$2(x_1+x_2+x_3)=0+1+1=2$

$\therefore \quad x_1+x_2+x_3=1$

Now substracing (iii), (v) and (i) respectively, we get

$x_1=0, x_2=0, x_3=1$

Again Adding (ii), (iv) and (vi)

$2(y_1+y_2+y_3)=1+1+0=2$

$\therefore \quad y_1+y_2+y_3=1$

Now subtracting (iv), (vi) and (ii) respectively we get,

$y_1=0, y_2=1, y_3=0$

$\therefore$ Co-ordinates of A, B and C will be

(0, 0), (0, 1) and (1, 0)

Question 29.
Show by section formula that the points (3, -2), (5, 2) and (8, 8) are collinear.
Solution:

Let the point (5, 2) divides the line joining the points (3, -2) and (8, 8) in the ratio of $m_1 : m_2$

$$\therefore 5 = \frac{m_1 \times 8 + m_2 \times 3}{m_1 + m_2} \Rightarrow 8m_1 + 3m_2 = 5m_1 + 5m_2$$

$$\Rightarrow 8\, m_1 - 5\, m_1 \Rightarrow 5\, m_2 - 3\, m_2$$

$$\Rightarrow \quad 3\, m_1 = 2\, m_2 \quad \Rightarrow \frac{m_1}{m_2} = \frac{2}{3} \qquad \ldots(i)$$

$$\text{Again } 2 = \frac{8m_1 - 2m_2}{m_1 + m_2}$$

$$\Rightarrow 8\, m_1 - 2\, m_2 = 2\, m_1 + 2\, m_2$$

$$\Rightarrow 8\, m_1 - 2\, m_1 = 2\, m_2 + 2\, m_2$$

$$\Rightarrow 6\, m_1 = 4\, m_2 \Rightarrow \frac{m_1}{m_2} = \frac{4}{6} = \frac{2}{3} \qquad \ldots(ii)$$

from (*i*) and (*ii*) it is clear that point (5, 2) lies on the line joining the points (3, – 2) and (8, 8). Hence proved.

Question 30.
Find the value of p for which the points (-5, 1), (1, p) and (4, -2) are collinear.

Solution:

Let points A (-5, 1), B (1, p) and C (4, -2)
are collinear and let point A (-5, 1) divides
BC in the ratio in $m_1 : m_2$

$$\therefore\ x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow -5 = \frac{m_1 \times 4 + m_1 \times 1}{m_1 + m_2} = \frac{4m_1 + m_2}{m_1 + m_2}$$

$$\Rightarrow -5m_1 - 5m_2 = 4m_1 + m_2$$

$$\Rightarrow -5m_1 - 4m_1 = m_2 + 5m_2$$

$$\Rightarrow -9m_1 = 6m_2 \Rightarrow \frac{m_1}{m_2} = \frac{6}{-9} = \frac{2}{-3} \qquad ....(i)$$

$$\text{and } \frac{m_1 \times (-2) + m_2 \times p}{m_1 + m_2} = \frac{-2m_1 + m_2 p}{m_1 + m_2}$$

$$\Rightarrow m_1 + m_2 = -2m_1 + m_2 p \Rightarrow m_1 + 2m_1 = m_2 p - m_2$$

$$\Rightarrow 3m_1 = m_2 (p - 1) \Rightarrow \frac{m_1}{m_2} = \frac{p-1}{3} \qquad ....(ii)$$

From (i) and (ii)

$$\frac{p-1}{3} = \frac{2}{-3} \Rightarrow -3p + 3 = 6$$

$$\Rightarrow -3p = 6 - 3 \Rightarrow -3p = 3 \Rightarrow \quad p = \frac{3}{-3} = -1$$

$$\therefore\ p = -1$$

Question 31.

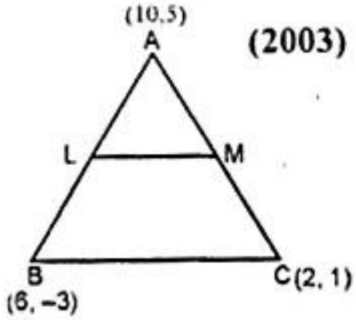
A (10, 5), B (6, -3) and C (2, 1) are the vertices of triangle ABC. L is the midpoint of AB, M is the mid-point of AC. Write down the co-ordinates of L and M. Show that LM = 12 BC.

Solution:

Co-ordinates of L will be

$$\left(\frac{10+6}{2}, \frac{5-3}{2}\right) or \left(\frac{16}{2}, \frac{2}{2}\right) or (8, 1)$$

(2003)



Co-ordinaes of M will be

$$= \left(\frac{10+2}{2}, \frac{5+1}{2}\right) \text{ or } = \left(\frac{12}{2}, \frac{6}{2}\right) \text{ or } (6, 3)$$

$$\text{Length of LM} = \sqrt{(6-8)^2 + (3-1)^2}$$

$$= \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$= \sqrt{4 \times 2} = 2\sqrt{2} \text{ units} \qquad \ldots(i)$$

$$\text{Length of BC} = \sqrt{(2-6)^2 + [1-(-3)]^2}$$

$$= \sqrt{(-4)^2 + (1+3)^2} = \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+6} = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2} \text{ units} \ldots(ii)$$

from (*i*) and (*ii*)

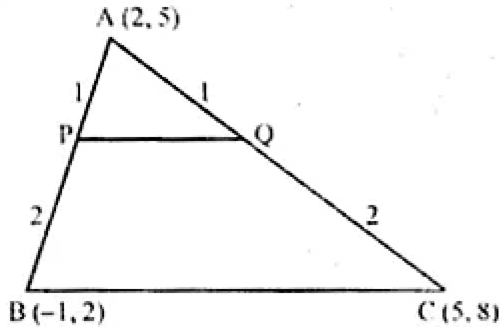
$$\text{LM} = \frac{1}{2} \text{ BC.}$$

Question 32.
A (2, 5), B (-1, 2) and C (5, 8) are the vertices of a triangle ABC. P and.Q are points on AB and AC respectively such that AP : PB = AQ : QC = 1 : 2.
(i) Find the co-ordinates of P and Q.
(ii) Show that PQ = 13 BC.
Solution:

A (2, 5), B (-1, 2) and C (5, 8) are the vertices of a ΔABC,

P and Q are points on AB

and AC respectively such that $\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$



Let co-ordinates of P be (x_1, y_1) and of Q be(x_2, y_2)

$\because$ P divides AB in the ratio 1 : 2

$$\therefore\ x_1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times (-1) + 2 \times 2}{1 + 2}$$

$$= \frac{-1+4}{3} = \frac{3}{3} = 1$$

$$y_1 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 5}{1+2} = \frac{2+10}{3} = \frac{12}{3} = 4$$

$\therefore$ Co-ordiantes of P will be (1, 4)
Similarly Q divides AC in the ratio 1 : 2

$$\therefore\ x_2 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\times5+2\times2}{1+2}$$

$$= \frac{5+4}{3} = \frac{9}{3} = 3$$

$$\text{and } y_2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times8+2\times5}{1+2}$$

$$= \frac{8+10}{3} = \frac{18}{3} = 6$$

$\therefore$ Co-ordinates of Q will be (3, 6)

(*ii*) Now iength of BC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(5+1)^2 + (8-2)^2} = \sqrt{(6)^2 + (6)^2}$$

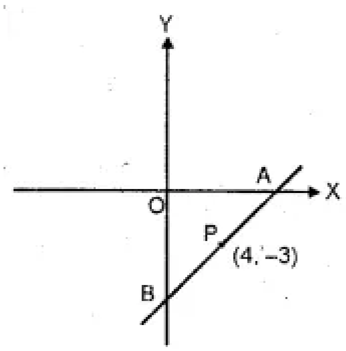
$$= \sqrt{36+36} + \sqrt{72} = \sqrt{2\times36} = 6\sqrt{2}$$

$$\text{and PQ} = \sqrt{(1-3)^2 + (4-6)^2}$$

$$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{2\times4} = 2\sqrt{2}$$

$$= \frac{3}{3} \times 2\sqrt{6} = \frac{6\sqrt{2}}{3} = \frac{BC}{3} = \frac{1}{3}BC$$

Question 33.
The mid-point of the line segment AB shown in the adjoining diagram is (4, -3). Write down die co-ordinates of A and B.



Solution:

A lies on x-axis and B on the y-axis.

Let co-ordinates of A be (x, 0) and of B be (0, y)

P (4, -3) is the mid-point of AB

$$\therefore\ 4 = \frac{x+0}{2} \Rightarrow x = 8$$

$$\text{and } -3 = \frac{0+y}{2} \Rightarrow y = -6$$

Co-ordinates of A will be (8, 0) and of B will be (0, – 6)

Question 34.

Find the co-ordinates of the centroid of a triangle whose vertices are A (-1, 3), B(1, -1) and C (5, 1) (2006)

Solution:

Co-ordinates of the centroid of a triangle,

whose vertices are (x1, y1), (x2, y2) and

$$(x_3, y_3) \text{ are } \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$\therefore$ Co-ordinates of the centroid of the given triangle

$$\text{are } \left(\frac{-1+1+5}{3}, \frac{3-1+1}{3}\right) \text{ i.e. } \left(\frac{5}{3}, 1\right)$$

Question 35.

Two vertices of a triangle are (3, -5) and (-7, 4). Find the third vertex given that the centroid is (2, -1).

Solution:

Let the co-ordinates of third vertices be (x, y)

and other two vertices are (3, -5) and (-7, 4)

and centroid = (2, -1).

$$\therefore\ 2 = \frac{3-7+x}{3} \Rightarrow \frac{x-4}{3} = 2$$

$$x - 4 = 6 \Rightarrow x = 6 + 4 \Rightarrow x = 10$$

$$\text{and} \Rightarrow -1 = \frac{-5+4+y}{3} \Rightarrow -3 = -1 + y$$

$$\Rightarrow y = -3 + 1 = 2$$

$\therefore$ Co-ordinates are (10, – 2)

Question 36.
The vertices of a triangle are A (-5, 3), B (p, -1) and C (6, q). Find the values of p and q if the centroid of the triangle ABC is the point (1, -1).
Solution:

The vertices of ΔABC are A (-5, 3), B (p, -1), C (6, q)

and the centroid of ΔABC is O (1, -1)

co-ordinates of the centroid of ΔABC will be

$$\left[\frac{-5+p+6}{3}, \frac{3-1+q}{3}\right] \Rightarrow \left(\frac{1+p}{3}, \frac{2+q}{3}\right)$$

But centroid is given (1, –1)

$\therefore$ Comparing, we get

$$\frac{1+p}{3} = 1 \Rightarrow 1 + p = 3$$

$$\Rightarrow p = 3 - 1 = 2$$

$$\text{and } \frac{2+q}{3} = -1 \Rightarrow 2 + q = -3$$

$$\Rightarrow q = -3 - 2 \Rightarrow q = -5$$

Hence $p = 2, q = -5$