Chapter 9 Arithmetic and Geometric Progressions Ex 9.3

Question 1.
Find the sum of the following A.P.s:
(i) 2, 7, 12, … to 10 terms
(ii) 115,112,110,… t0 11 terms
Solution:

(i) 2, 7, 12, ... to 10 terms

Here a = 2, d = 7 − 2 = 5 and n = 10

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 \times 2 + (10-1) \times 5]$$

$$= 5(4 + 45) = 5 \times 49 = 245$$

(*ii*) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms

$$a = \frac{1}{15},$$

$$d = \frac{1}{12} - \frac{1}{15}$$

$$= \frac{5-4}{60} = \frac{1}{60} \text{ or } \frac{1}{10} - \frac{1}{12}$$

$$= \frac{6-5}{60} = \frac{1}{60}$$

$$n = 11$$

$$\therefore S_{11} = \frac{n}{2} \times [2a + (n-1)d]$$

$$= \frac{11}{2} \times \left[2 \times \frac{1}{15} + (11-1) \times \frac{1}{60}\right]$$

$$= \frac{11}{2} \times \left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2} \times \left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2} \times \frac{9}{30} = \frac{33}{20} \text{ or}$$

$$= 1\frac{13}{20}$$

Question 2.
How many terms of the A.P. 27, 24, 21, …, should be taken so that their sum is zero?
Solution:
A.P. = 27, 24, 21,...

a = 27

d = 24 − 27 = -3

$S_n = 0$

Let n terms be there in A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 0 = \frac{n}{2}[(2 \times 27) + (n-1)(-3)]$$

$$\Rightarrow 0 = n[54 - 3n + 3]$$

$$\Rightarrow n[57 - 3n] = 0$$

$$\Rightarrow (57 - 3n) = \frac{0}{n} = 0$$

$$\Rightarrow 3n = 57$$

$$\therefore n = \frac{57}{3} = 19$$

Question 3.
Find the sums given below :
(i) 34 + 32 + 30 + … + 10
(ii) -5 + (-8) + (-11) + … + (-230)
Solution:

(i) 34 + 32 + 30 + ... + 10

Here, a = 34, d = 32 − 34 = -2, l = 10

$T_n = a + (n - 1)d$

10 = 34 + (n − 1)(-2)

-24 = -2 (n − 1)

$$= \frac{-24}{-2} = n - 1 \Rightarrow n - 1 = 12$$

$$\therefore\ n = 12 + 1 = 13$$

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{13}{2}[34 + 10] = \frac{13}{2} \times 44 = 286$$

(*ii*) $-5 + (-8) + (-11) + \ldots + (-230)$

Here, $a = -5$, $d = -8 - (-5) = -8 + 5 = -3$

$l = -230$

$$\therefore\ l = a + (n - 1)d \Rightarrow -230 = -5 + (n - 1)(-3)$$

$$-230 + 5 = -3(n - 1) \Rightarrow -225 = -3(n - 1)$$

$$\frac{-225}{-3} = n - 1 \Rightarrow n - 1 = 75$$

$$\Rightarrow n = 75 + 1 = 76$$

$$\therefore\ S_n = \frac{n}{2}[a + l] = \frac{76}{2}[-5 + (-230)]$$

$$= 38[-5 - 230] = 38 \times (-235) = -8930$$

Question 4.
In an A.P. (with usual notations) :
(i) given a = 5, d = 3, a_n = 50, find n and S_n
(ii) given a = 7, a_{13} = 35, find d and S_{13}
(iii) given d = 5, S_9 = 75, find a and a_9
(iv) given a = 8, a_n = 62, S_n = 210, find n and d
(v) given a = 3, n = 8, S = 192, find d.
Solution:

(i) $a = 5$, $d = 3$, $a_n = 50$

$a_n = a + (n - 1)d$

$50 = 5 + (n - 1) \times 3$

$\Rightarrow 50 - 5 = 3(n - 1)$

$$\Rightarrow 45 = 3(n-1) \Rightarrow \frac{45}{3} = n - 1$$

$$\Rightarrow n - 1 = 15 \Rightarrow n = 15 + 1 = 16$$

$$\therefore\ n = 16$$

$$\text{and } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{16}{2}[2 \times 5 + (16 - 1) \times 3] = 8[10 + 45]$$

$$= 8 \times 55 = 440$$

(*ii*) $a = 7, a_{13} = 35$

$a_n = a + (n-1)d$

$35 = 7 + (13-1)d \Rightarrow 35 - 7 = 12d$

$$\Rightarrow 28 = 12d \Rightarrow d = \frac{28}{12} = \frac{7}{3} = 2\frac{1}{3}$$

and $S_{13} = \frac{n}{2}\cdot[2a + (n-1)d]$

$$= \frac{13}{2}\left[2\times7+(13-1)\times\frac{7}{3}\right]$$

$$= \frac{13}{2}[14 + 28] = \frac{13}{2} \times (42)$$

$= 13 \times 21 = 273$

(*iii*) $d = 5, S_9 = 75$

$a_n = a + (n-1)d$

$a_9 = a + (9-1) \times 5$

$= a + 40$ .... (i)

$$S_9 = \frac{n}{2}[2a + (n-1)d]$$

$$75 = \frac{9}{2}[2a + 8 \times 5]$$

$$\frac{150}{9} = 2a + 40$$

$$2a = \frac{150}{9} - 40 = \frac{50}{3} - 40$$

$$2a = \frac{-70}{3} \Rightarrow a = \frac{-70}{2 \times 3}$$

$$a = \frac{-35}{3}$$

From (*i*),

$$a_9 = a + 40 = \frac{-35}{3} + 40$$

$$= \frac{-35+120}{3} = \frac{85}{3}$$

$$\therefore\ a = \frac{-35}{3},\ a_9 = \frac{85}{3}$$

(*iv*) $a = 8,\ a_n = 62,\ S_n = 210$

$$a_n = a + (n-1)d$$

$$62 = 8 + (n-1)d$$

$$(n-1)d = 62 - 8 = 54 \qquad \ldots(i)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$210 = \frac{n}{2}[2 \times 8 + 54] \qquad \text{[From } (i)\text{]}$$

$$420 = n(16 + 54) \Rightarrow 420 = 70n$$

$$n = \frac{420}{70} = 6$$

$$\therefore\ (6-1)d = 54$$

$$\Rightarrow 5d = 54$$

$$\Rightarrow d = \frac{54}{5}$$

Hence $d = \dfrac{54}{5}$ and $n = 6$

(*v*) $a = 3,\ n = 8,\ S = 192$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$192 = \frac{8}{2}[2 \times 3 + 7 \times d]$$

$$192 = 4[6 + 7d] \Rightarrow \frac{192}{4} = 6 + 7d$$

$$\Rightarrow 48 = 6 + 7d \Rightarrow 7d = 48 - 6 = 42$$

$$d = \frac{42}{7} = 6$$

$$\therefore\ d = 6$$

Question 5.
(i) The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
(ii) The sum of the first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term.
Solution:

(i) First term of an A.P. (a) = 5

Last term (l) = 45

Sum = 400

l = a + (n − 1)d

45 = 5 + (n − 1)d

⇒ (n − 1)d = 45 − 5 = 40 ...(i)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$400 = \frac{n}{2}[2 \times 5 + 40] \Rightarrow 800 = n(10 + 40)$$

$$50n = 800 \Rightarrow n = \frac{800}{50} = 16$$

From (*i*),

$$(16 - 1)d = 40 \Rightarrow 15d = 40 \Rightarrow d = \frac{40}{15}$$

$$\therefore\ d = \frac{8}{3} \text{ and } n = 16$$

(*ii*) Let a be the first term and d be the common difference.

Now, $a = 15$

Sum of first n terms of an AP is given by,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{15} = \frac{15}{2}[2a + (15-1)d]$$

$$\Rightarrow 750 = \frac{15}{2}(2a + 14d)$$

$\Rightarrow a + 7d = 50$

$\Rightarrow 15 + 7d = 50$

$\Rightarrow 7d = 35$

$\Rightarrow d = 5$

Now, 20th term $= a_{20} = a + 19d$

$= 15 + 19 \times 5 = 15 + 95 = 110$

Question 6.

The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:
First term of an A.P. (a) = 17
and last term (l) = 350
d= 9
$l = T_n = a + (n - 1)d$

$$350 = 17 + (n - 1) \times 9$$
$$\Rightarrow 350 - 17 = 9(n - 1)$$
$$\Rightarrow 333 = 9(n - 1) \Rightarrow n - 1 = \frac{333}{9} = 37$$
$$n = 37 + 1 = 38$$
$$\text{and } S_n = \frac{n}{2}[2a + (n - 1)d]$$
$$= \frac{38}{2}[2 \times 17 + (38 - 1) \times 9]$$
$$= 19[34 + 37 \times 9] = 19\,[34 + 333]$$
$$= 19 \times 367 = 6973$$

Hence $n = 38$ and $S_n = 6973$

Question 7.
Solve for x : 1 + 4 + 7 + 10 + … + x = 287.

Solution:

$1 + 4 + 7 + 10 + \ldots + x = 287$

Here, $a = 1$, $d = 4 - 1 = 3$, $n = x$

$l = x = a = (n - 1)d = 1 + (n - 1) \times 3$

$\Rightarrow x - 1 = (n - 1)d$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$287 = \frac{n}{2}[2 \times 1 + (n - 1)3]$$

$574 = n(2 - 3n - 3)$

$\Rightarrow 3n^2 - n - 574 = 0$

$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow 3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow (n - 14)(3n + 41) = 0$

Either $n - 14 = 0$, then $n = 14$

or $3n + 41 = 0$, then $3n = -41 \Rightarrow n = \frac{-41}{3}$

which is not possible being negative.

$\therefore n = 14$

Now, $x = a + (n - 1)d$

$= 1 + (14 - 1) \times 3 = 1 + 13 \times 3$

$= 1 + 39 = 40$

$\therefore x = 40$

Question 8.
(i) How many terms of the A.P. 25, 22, 19, … are needed to give the sum 116? Also, find the last term.
(ii) How many terms of the A.P. 24, 21, 18, … must be taken so that the sum is 78? Explain the double answer.
Solution:

(i) A.P. is 25, 22, 19, ...

Sum = 116

Here, a = 25, d = 22 − 25 = -3

Let number of terms be n, then

$$116 = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow 232 = n[2 \times 25 + (n-1)(-3)]$

$\Rightarrow 232 = n[50 - 3n + 3] \Rightarrow 232 = n(53 - 3n)$

$\Rightarrow 232 = 53n - 3n^2$

$\Rightarrow 3n^2 - 53n + 232 = 0$

$$\left\{\begin{array}{l} \because 232 \times 3 = 696 \\ \therefore 696 = -24 \times (-29) \\ \quad -53 = -24 - 29 \end{array}\right\}$$

$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$

$\Rightarrow 3n(n-8) - 29(n-8) = 0$

$\Rightarrow (n-8)(3n-29) = 0$

Either $n - 8 = 0$, then $n = 8$

or $3n - 29 = 0$, then $3n = 29 \Rightarrow n = \frac{29}{3}$

which is not possible because of fractioin

$\therefore\ n = 8$

Now, $T = a + (n - 1)d$

$= 25 + 7 \times (-3) = 25 - 21 = 4$

(*ii*) A.P. is 24, 21, 18, ...

Sum = 78

Here, $a = 24$, $d = 21 - 24 = -3$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 78 = \frac{n}{2}[2 \times 24 + (n - 1)(-3)]$$

$\Rightarrow 156 = n(48 - 3n + 3)$

$\Rightarrow 156 = 51n - 3n^2$

$\Rightarrow 3n^2 - 51n + 156 = 0$

$\Rightarrow 3n^2 - 12n - 39n + 156 = 0$

$$\left\{\begin{array}{l} \because 156 \times 3 = 468 \\ \therefore 468 = -12 \times -39 \\ \quad -51 = -12 - 39 \end{array}\right\}$$

$\Rightarrow 3n(n-4) - 39(n-4) = 0$

$\Rightarrow (n-4)(3n-39) = 0$

Either $n - 4 = 0$, then $n = 4$

or $3n - 39 = 0$, then $3n = 39 \Rightarrow n = 13$

$\therefore n = 4$ and 13

$n_4 = a + (n-1)d = 24 + 3(-3)$

$= 24 - 9 = 15$

$n_{13} = 24 + 12(-3) = 24 - 36 = -12$

$\therefore$ Sum of 5th term to 13 term = 0

($\because 12 + 9 + 6 + 3 + 0 + (-3) + (-6) + (-9) + (-12) = 0$

Question 9.
Find the sum of first 22 terms, of an A.P. in which d = 7 and a_{22} is 149.
Solution:

Sum of first 22 terms of an A.P. whose d = 7

$a_{22} = 149$ and n = 22

$149 = a + (n-1)d = a + 21 \times 7$

$149 = a + 147 \Rightarrow a = 149 - 147 = 2$

$$\therefore S_{22} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{22}{2}[2 \times 2 + (22-1)(7)]$$

$= 11[4 + 21 \times 7] = 11 \times [4 + 147]$

$= 11 \times 151 = 1661$

Question 10.
(i) Find the sum of the first 51 terms of the A.P. whose second and third terms are 14 and 18 respectively.
(ii) If the third term of an A.P. is 1 and 6th term is -11, find the sum of its first 32 terms.
Solution:

Sum of first 51 terms of an A.P. in which

$T_2 = 14, T_3 = 18$

$\therefore\ d = T_3 - T_2 = 18 - 14 = 4$

and $a = T_1 = 14 - 4 = 10,\ n = 51$

Now, $S_{51} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 50 \times 4] = \frac{51}{2}[20 + 200]$

$= \frac{51}{2} \times 220 = 5610$

(*ii*) $T_3 = 1,\ T_6 = -11,\ n = 32$

$$a + 2d = 1 \quad \ldots(i)$$

$$a + 5d = -11 \quad \ldots(ii)$$

$$- \quad - \qquad +$$

Subtracting (*i*) and (*ii*),

$-3d = 12 \Rightarrow d = \frac{12}{-3} = -4$

Substitute the value of d in eq. (*i*)

$a + 2(-4) = 1 \Rightarrow a - 8 = 1$

$a = 1 + 8 = 9$

$\therefore\ a = 9,\ d = -4$

$S_{32} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{32}{2}[2 \times 9 + (32 - 1) \times (-4)]$

$= 16[18 + 31 \times (-4)]$

$= 16[18 - 124] = 16 \times (-106)] = -1696$

Question 11.
If the sum of the first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms.
Solution:

$S_6 = 36$

$S_{16} = 256$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore\ S_6 = \frac{6}{2}[2a + (6-1)d] = 36$$

$\Rightarrow 3[2a + 5d] = 36$

$\Rightarrow 2a + 5d = 12$ ...(*i*)

and $S_{16} = \frac{16}{2}[2a + (16-1)d] = 256$

$8[2a + 15d] = 256$

$2a + 15d = 32$ ...(*ii*)

$$\Rightarrow \quad \begin{array}{r} 2a + 5d = 12 \\ 2a + 15d = 32 \\ - \quad - \qquad - \end{array}$$

Subtracting (*i*) from (*ii*),

$$-10d = -20 \Rightarrow d = \frac{-20}{-10} = 2$$

Substitute the value of d in eq. (*i*),

$2a + 5d = 12 \Rightarrow 2a + 5 \times 2 = 12$

$\Rightarrow 2a + 10 = 12 \Rightarrow 2a = 12 - 10 = 2$

$$\Rightarrow a = \frac{2}{2} = 1$$

$\therefore\ a = 1, d = 2$

Now, $S_{10} = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{10}{2}[2 \times 1 + (10-1) \times 2]$$

$= 5[2 + 9 \times 2] = 5[2 + 18]$

$= 5 \times 20 = 100$

Question 12.
Show that $a_1, a_2, a_3, \ldots$ form an A.P. where a_n is defined as $a_n = 3 + 4n$. Also, find the sum of the first 15 terms.
Solution:

$a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

$a_4 = 3 + 4 \times 4 = 3 + 16 = 19$

and so on Here, a = 1 and d = 11 − 7 = 4

$$S_{15} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{15}{2}[2 \times 7 + (15-1) \times 4]$$

$$= \frac{15}{2}[14 + 14 \times 4] = \frac{15}{2}[14 + 56]$$

$$= \frac{15}{2} \times 70 = 525$$

Question 13.
(i) If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an A.P. Also find S_{20}.
(ii) Find the common difference of an A.P. whose first term is 5 and the sum of the first four terms is half the sum of the next four terms.
Solution:

(i) $a_n = 3 - 4n$

$a_1 = 3 - 4 \times 1 = 3 - 4 = -1$

$a_2 = 3 - 4 \times 2 = 3 - 8 = -5$

$a_3 = 3 - 4 \times 3 = 3 - 12 = -9$

$a_4 = 3 - 4 \times 4 = 3 - 16 = -13$ and so on

Here, $a = -1$, $d = -5 - (-1) = -5 + 1 = -4$

Now, $S_{20} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2 \times (-1) + (20 - 1) \times (-4)]$

$= 10[-2 + 19 \times (-4)]$

$= 10[-2 - 76] = 10 \times (-78) = -780$

(*ii*) Let a and d be the first term and common difference of A.P. respectively.

Given, $a = 5$

$$a_1 + a_2 + a_3 + a_4 = \frac{1}{2}(a_5 + a_6 + a_7 + a_8)$$

$\therefore\ a + (a + d) + (a + 2d) + (a + 3d) = \frac{1}{2}[(a + 4d) + (a + 5d) + (a + 6d) + (a + 7d)]$

$\Rightarrow 2(4a + 6d) = (4a + 22d)$

$\Rightarrow 2(20 + 6d) = (20 + 22d) \qquad (\because a = 5)$

$\Rightarrow 40 + 12d = 20 + 22d$

$\Rightarrow 10d = 20$

$\Rightarrow d = 2$

Thus, the common difference of A.P. is 2.

Question 14.

The sum of first n terms of an A.P. whose first term is 8 and the common difference is 20 equal to the sum of first 2n terms of another A.P. whose first term is -30 and the common difference is 8. Find n.

Solution:

In an A.P.

$S_n = S_{2n}$

For the first A.P. a = 8, d = 20

and for second A.P. a = -30, d = 8

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 8 + (n-1) \times 20]$$

$$= \frac{n}{2}[16 + 20n - 20] = \frac{n}{2}[20n - 4]$$

$$= 10n^2 - 2n \qquad \ldots(i)$$

Similarly,

$$S_{2n} = 2\frac{n}{2}[2a + (2n-1)d]$$

$$= n[2 \times (-30) + (2n-1) \times 8]$$

$$= n[-60 + 16n - 8] = n(16n - 68)$$

$$= 16n^2 - 68n$$

$$\because S_n = 2_{2n}$$

$$\therefore 10n^2 - 2n = 16n^2 - 68n$$

$$\Rightarrow 16n^2 - 10n^2 - 68n + 2n = 0$$

$$\Rightarrow 6n^2 - 66n = 0 \Rightarrow n^2 - 11n = 0$$

$$n(n - 11) = 0$$

Either $n = 0$ which is not possible

or $n - 11 = 0$, then $n = 11$

$$\therefore n = 11$$

Question 15.
The sum of the first six terms of an arithmetic progression is 42. The ratio of the 10th term to the 30th term is 13. Calculate the first and the thirteenth term.

Solution:

$T_{10} : T_{30} = 1 : 3, S_6 = 42$

Let a be the first term and d be a common difference, then

$$\frac{a+9d}{a+29d} = \frac{1}{3} \Rightarrow 3a + 27d = a + 29d$$

$$\Rightarrow 3a - a = 29d - 27d$$

$$\Rightarrow 2a = 2d \Rightarrow a = d$$

$$\text{Now, } S_6 = 42 = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 42 = \frac{6}{2}[2a + (6-1)d]$$

$$\Rightarrow 42 = 3[2a + 5d]$$

$$\Rightarrow 14 = 2a + 5d \Rightarrow 14 = 2a + 5a \quad (\because d = a)$$

$$\Rightarrow 7a = 14 \Rightarrow a = \frac{14}{7} = 2$$

$$\therefore a = d = 2$$

Now, $T_{13} = a + (n-1)d$

$= 2 + (13-1) \times 2 = 2 + 12 \times 2$

$= 2 + 24 = 26$

$\therefore$ 1st term is 2 and thirteenth term is 26

Question 16.
In an A.P., the sum of its first n terms is $6n - n^2$. Find the 25th term.

Solution:

$S_n = 6n - n^2$

$T_{25} = ?$

$S_{(n-1)} = 6(n-1) - (n-1)^2$

$= 6n - 6 - (n^2 - 2n + 1)$

$= 6n - 6 - n^2 + 2n - 1 = 8n - n^2 - 7$

$a_n = S_n - S_{n-1}$

$= 6n - n^2 - 8n + n^2 + 7$

$= -2n + 7$

$a_{25} = -2(25) + 7 = -50 + 7 = -43$

Question 17.

If the sum of first n terms of an A.P. is $4n - n^2$, what is the first term (i. e. S_1)? What is the sum of the first two terms? What is the second term? Also, find the 3rd term, the 10th term, and the nth terms?

Solution:

$S_n = 4n - n^2$

$S_n - 1 = 4(n-1) - (n-1)^2$

$= 4n - 4 - (n^2 - 2n + 1)$

$= 4n - 4 - n^2 + 2n - 1 = 6n - n^2 - 5$

$\therefore\ a_n = S_n - S_{n-1} = 4n - n^2 - (6n - n^2 - 5)$

$= 4n - n^2 - 6n + n^2 + 5$

$= -2n + 5$

$a_1 = -2 \times 1 + 5 = -2 + 5 = 3$

$a_2 = -2 \times 2 + 5 = -4 + 5 = 1$

$a_3 = -2 \times 3 + 5 = -6 + 5 = -1$

$a_4 = -2 \times 4 + 5 = -8 + 5 = -3$

$a_{10} = -2 \times 10 + 5 = -20 + 5 = -15$

$S_2 = 4n - n^2 = 4 \times 2 - (2)^2$

$= 8 - 4 = 4$

Hence, $S_2 = 4$, $a_1 = 1$, $a_3 = -1$, $a_{10} = -15$

$a_n = -2n + 5$

Question 18.

If S_n denotes the sum of first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.

Solution:

S_n denotes the sum of first n terms of an A.P.

To prove: $S_{30} = 3(S_{20} - S_{10})$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore\ S_{10} = \frac{10}{2}[2a + (10-1)d] = 5(2a + 9d)$$

$$= 10a + 45d$$

$$S_{20} = \frac{20}{2}[2a + (20-1)d] = 10(2a + 19d)$$

$$= 20a + 190d$$

$$S_{30} = \frac{30}{2}[2a + (30-1)d = 15(2a + 29d)$$

$$= 30a + 435d$$

Now,

R.H.S. $= 3(S_{20} - S_{10})$

$= 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d]$

$= 30a + 435d$

$= S_{30} =$ L.H.S.

Question 19.

(i) Find the sum of the first 1000 positive integers.

(ii) Find the sum of first 15 multiples of 8.

Solution:

(i) Sum of first 1000 positive integers

i. e., 1 + 2 + 3+ 4 + ... + 1000

Here, a = 1, d = 1, n = 1000

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{1000}{2}[2 \times 1 + (1000 - 1)1]$$

$$= 500[2 + 999] = 500 \times 1001$$

$$= 500500$$

(ii) Sum of first 15 multiples of 8

8 + 16 + 24 + 32 + ... 120

Here, $a = 8, d = 8, n = 15$

$$\therefore S_{15} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{15}{2}[2 \times 8 + (15 - 1) \times 8]$$

$$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 15 \times 64 = 960$$

Question 20.

(i) Find the sum of all two digit natural numbers which are divisible by 4.

(ii) Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

(iii) Find the sum of all multiples of 9 lying between 300 and 700.

(iv) Find the sum of all natural numbers less than 100 which are divisible by 6.

Solution:

(i) Sum of two digit natural numbers which are divisible by 4

which are 12, 16, 20, 24, ..., 96

Here, a = 12, d = 16 − 12 = 4, l = 96

Here, $a = 12$, $d = 16 - 12 = 4$, $l = 96$

$\therefore\ l = 96 = a + (n-1)d = 12 + (n-1) \times 4$

$96 = 12 + 4n - 4 = 8 + 4n$

$$\Rightarrow 4n = 96 - 8 = 88 \Rightarrow n = \frac{88}{4} = 22$$

$$\therefore\ S_{22} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{22}{2}[2 \times 12 + (22-1) \times 4]$$

$= 11[24 + 21 \times 4] = 11[24 + 84]$

$= 11 \times 108 = 1188$

(*ii*) Sum of all natural numbers between 100 and 200 which are divisible by 4 which are

104, 108, 112, 116, ..., 196

Here, $a = 104$, $d = 108 - 104 = 4$, $l = 196$

$l = a_n = 196 = a + (n-1)d$

$\Rightarrow 196 = 104 + (n-1) \times 4$

$196 - 104 = (n-1)4 \Rightarrow 92 = (n-1)4$

$$n - 1 = \frac{92}{4} = 23$$

$\therefore\ n = 23 + 1 = 24$

$$\text{Now, } S_{24} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{24}{2}[2 \times 104 + (24-1) \times 4]$$

$= 12[208 + 23 \times 4] = 12 \times [208 + 92]$

$= 12 \times 300 = 3600$

(*iii*) Sum of all natural numbers multiple of 9 lying between 300 and 700 which are

306, 315, 324, 333, ..., 693

Here, $a = 306$, $d = 9$, $l = 693$

$l = a_n = 693 = a + (n - 1)d$

$= 306 + (n - 1) \times 9$

$693 - 306 = 9(n - 1)$

$$387 = 9(n - 1) \Rightarrow n - 1 = \frac{387}{9} = 43$$

$\therefore\ n = 43 + 1 = 44$

$$S_{44} = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{44}{2}[2 \times 306 + (44 - 1) \times 9]$$

$= 22[612 + 43 \times 9]$

$= 22[612 + 387] = 22 \times 999 = 21978$

$$\begin{array}{r} 999 \\ \times 22 \\ \hline 1998 \\ 1998\times \\ \hline 21978 \\ \hline \end{array}$$

(*iv*) Sum of all natural numbers less then 100 which are divisible by 6 which are

6, 12, 18, 24, ..., 96

Here, $a = 6, d = 6, l = 96$

$a_n = l = 96 = a + (n - 1)d$

$\Rightarrow 96 = 6 + (n - 1) \times 6$

$96 - 6 = 6(n - 1)$

$\frac{90}{6} = n - 1 \Rightarrow n - 1 = 15$

$n = 15 + 1 = 16$

$\therefore \; S_{16} = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{16}{2}[2 \times 6 + (16 - 1) \times 6]$

$= 8[12 + 15 \times 6] = 8[12 + 90]$

$= 8 \times 102 = 816$