Maths Chapter 8 Matrices Ex 8.3

Question 1.
If A = [345−2] and B = [24], is the product AB possible ? Give a reason. If yes, find AB.
Solution:

Yes, the product is possible because of

number of column in A = number of row in B

i.e., (2 x 2). (2 x 1) = (2 x 1) is the order of the matrix.

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 5 \times 4 \\ 4 \times 2 + (-2) \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

Question 2.
If A = [2153],B = [1−3−12], find AB and BA, Is AB = BA ?
Solution:

$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$

$$\therefore A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 15 & -2 + 10 \\ 1 - 9 & -1 + 6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

$$\text{and } B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 1 & 5 - 3 \\ -6 + 2 & -15 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence $AB \neq BA$.

Question 3.
If P = [426−8],Q = [2−1−31]
Find 2PQ

Solution:

P = $\begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}$,

Q = $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$

$$2PQ = 2\begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 8-6 & -12+6 \\ 4+8 & -6-8 \end{bmatrix} = 2\begin{bmatrix} 8 & -6 \\ 12 & -14 \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 24 & -28 \end{bmatrix}$$

Question 4.
Given A = [1813] , evaluate A² – 4A
Solution:

A = $\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$$A^2 - 4A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4 & 4-4 \\ 32-32 & 17-12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Question 5.
If A = [3274], B = [0523] and C = [1−4−56]
Find AB – 5C

Solution:

A = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

AB = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3\times0+7\times5 & 3\times2+7\times3 \\ 2\times0+4\times5 & 2\times2+4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

Question 6.
If A = [1221] and B = [2112], find A(BA)
Solution:

A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

B = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A\,(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} = \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

Question 7.
Given matrices:
A = [2412] and B = [3−14−2], C = [−301−2]
Find the products of (i) ABC (ii) ACB and state whether they are equal.

Solution:

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$

$C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

$$ABC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$$ACB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -18+0 & -24+0 \\ -36+0 & -48+0 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

$\therefore ABC \neq ACB.$

Question 8.

Evaluate : [4sin30ₒsin90ₒ2cos60ₒ2cos0ₒ][4554]

Solution:

$$\begin{bmatrix} 4\sin 30^o & 2cos60^o \\ sin90^o & 2cos0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$sin30^o = \frac{1}{2}, cos60^o = \frac{1}{2}$

$\sin 90^o = 1$ and $\cos 0^o = 1$

$$\therefore \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+1\times5 & 2\times5+1\times4 \\ 1\times4+2\times5 & 1\times5+2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Question 9.

If A = $[-1 2 3 4]$, B = $[2 -4 -3 -6]$ find the matrix AB + BA

Solution:

A = $\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$,

B = $\begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -2-12 & 3-18 \\ 4-16 & -6-24 \end{bmatrix} = \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-6 & 6-12 \\ 4-12 & -12-24 \end{bmatrix} = \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$$

$$\therefore AB + BA$$

$$= \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix} + \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$$

$$= \begin{bmatrix} -14-8 & -15-6 \\ -12-8 & -30-36 \end{bmatrix} = \begin{bmatrix} -22 & -21 \\ -20 & -66 \end{bmatrix}$$

Question 10.

A = [1324] and B = [6111], C = [−20−31]

find each of the following and state if they are equal.

(i) CA + B

(ii) A + CB

Solution:

(i) CA + B

$$CA = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-9 & -4-12 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix}$$

$$CA + B = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -11+6 & -16+1 \\ 3+1 & 4+1 \end{bmatrix} = \begin{bmatrix} -5 & -15 \\ 4 & 5 \end{bmatrix}.$$

(*ii*) A + CB

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -12-3 & -2-3 \\ 0+1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-15 & 2-5 \\ 3+1 & 4+1 \end{bmatrix} = \begin{bmatrix} -14 & -3 \\ 4 & 5 \end{bmatrix}$$

We can say that $CA + B \neq A + CB$.

Question 11.

If A = [12−2−1] and B = [3−221]

Find 2B – A^2

Solution:

$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

$2B = 2\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & -2+2 \\ 2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\therefore\ 2B - A^2 = \begin{bmatrix} 6 & 4 \\ -4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-3) & 4-0 \\ -4-0 & 2-(-3) \end{bmatrix} = \begin{bmatrix} 6+3 & 4 \\ -4 & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ -4 & 5 \end{bmatrix}$$

Question 12.
If A = [1324] and B = [2412], C = [5714], compute
(i) A(B + C)
(ii) (B + C)A

Solution:

(i) A(B + C)

A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

B = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$,

C = $\begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$

$$A(B + C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left[\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2+5 & 1+1 \\ 4+7 & 2+4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7+22 & 2+12 \\ 21+44 & 6+24 \end{bmatrix} = \begin{bmatrix} 29 & 14 \\ 65 & 30 \end{bmatrix}$$

$$(B + C)A = \begin{bmatrix} 7 & 2 \\ 11 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7+6 & 14+8 \\ 11+18 & 22+24 \end{bmatrix} = \begin{bmatrix} 13 & 22 \\ 29 & 46 \end{bmatrix}$$

Question 13.
If A = [1223] and B = [2312], C = [1331]
find the matrix C(B – A)

Solution:

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

$$B - A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$C(B - A) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & -1-3 \\ 3+1 & -3-1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}$$

Question 14.

A = [1201] and B = [2−130]

Find $A^2 + AB + B^2$

Solution:

Given that

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times2 & 1\times0+0\times1 \\ 2\times1+1\times2 & 2\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+0\times-1 & 1\times3+0\times0 \\ 2\times2+1\times-1 & 2\times3+1\times0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$\mathbf{B^2 = B \times B} = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times(-1) & 2\times3+3\times0 \\ -1\times2+0\times(-1) & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$A^2+AB+B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+1 & 0+3+6 \\ 4+3-2 & 1+6+-3 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

Question 15.
If A = $[2 0 1 -2]$ and B = $[4 -3 1 -2]$, C = $[-3 -1 2 4]$
Find $A^2 + AC - 5B$
Solution:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$
$$B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix},$$
$$C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$A^2 + AC - 5B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

(Subsitituting the values from question)

$$= \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} + \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} - 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

Question 16.
If A = $[1 0 0 -1]$, find A2 and A3.Also state that which of these is equal to A

Solution:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

From above, it is clear that $A^3 = A$.

Question 17.

If X = [4−112], show that 6X – X² = 9I Where I is the unit matrix.

Solution:

Given that

$$X = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$x^2 = x \times x = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$\text{L.H.S. } 6X - X^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24-15 & 6-6 \\ -6-6 & 12-3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I = \text{R.H.S.}$$

Hence proved.

Question 18.

Show that [1221] is a solution of the matrix equation X² – 2X – 3I = 0,Where I is the unit matrix of order 2

Solution:

Given

$X^2 - 2X - 3I = 0$

Solution = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

or

$X = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\therefore X^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Now $X^2 - 2X - 3I$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4+0 \\ 4-4-0 & 5-2-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore X^2 - 2X - 3I = 0$ Hence proved.

Question 19.
Find the matrix X of order 2 × 2 which satisfies the equation
[3274][0523]+2X=[1−4−56]

Solution:

Given

$$\begin{bmatrix}3 & 7\\2 & 4\end{bmatrix}\begin{bmatrix}0 & 2\\5 & 3\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\-4 & 6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0+35 & 6+21\\0+20 & 4+12\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\-4 & 6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}35 & 27\\20 & 16\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\-4 & 6\end{bmatrix}$$

$$2X=-\begin{bmatrix}35 & 27\\20 & 16\end{bmatrix}+\begin{bmatrix}1 & -5\\-4 & 6\end{bmatrix}=\begin{bmatrix}-34 & -32\\-24 & -10\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}-34 & -32\\-24 & -10\end{bmatrix}=\begin{bmatrix}-17 & -16\\-12 & -5\end{bmatrix}$$

Question 20.

If A = $[1x1x]$, find the value of x, so that A^2 – 0

Solution:

Given

$$A=\begin{bmatrix}1 & 1\\x & x\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 1\\x & x\end{bmatrix}\begin{bmatrix}1 & 1\\x & x\end{bmatrix}$$

$$=\begin{bmatrix}1+x & 1+x\\x+x^2 & x+x^2\end{bmatrix}$$

$$\because \quad A^2=0$$

$$\therefore =\begin{bmatrix}1+x & 1+x\\x+x^2 & x+x^2\end{bmatrix}=0=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Comparing $1+x=0 \Rightarrow x=-1$

Question 21.

If $[1030][2-1]=[x0]$ Find the value of x

Solution:

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

Comparing the corresponding elements

x = -1

Question 22.

(i) Find x and y if [−302−5][x2]=[−5y]

(ii) Find x and y if [2xyx3y][32]=[169]

Solution:

(i) $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -3x & 4 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} -3x & +4 \\ & -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

Comparing the corresponding elements

$-3x + 4 = -5 \Rightarrow -3x = -5 - 4 = -9$

$\therefore x = \frac{-9}{-3} = 3$

$-10 = y \Rightarrow y = -10$

Hence $x = 3, y = -10$

$$(ii) \quad \begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \times 3 + x \times 2 \\ y \times 3 + 3y \times 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

Comparing, we get

$$8x = 16 \Rightarrow x = \frac{16}{8} = 2$$

$$\text{and } 9y = 9 \Rightarrow y = \frac{9}{9} = 1$$

Here x = 2, y = 1

Question 23.
Find x and y if
[x+y2xyx−y][2−1]=[32]

Solution:

Given

$$\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x+2y & -y \\ 4x & -x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & +y \\ 3x & +y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Comparing the corresponding elements

$2x + y = 3$ ...(*i*)

$3x + y = 2$ ...(*ii*)

Subtracting, we get

$-x = 1 \Rightarrow x = -1$

Substituting the value of x in (*i*)

$2(-1) + y = 3 \Rightarrow -2 + y = 3$

$\Rightarrow \quad y = 3 + 2 = 5$

Hence $x = -1, y = 5$

Question 24.

If [1323][x00y]=[x900] find the values of x and y

Solution:

Given

$$\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x+0 & 0+2y \\ 3x+0 & 0+3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x & 2y \\ 3x & 3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

Comparing the corresponding elements

$2y = 0 \Rightarrow y = 0$

$3x = 9 \Rightarrow x = 3$

Hence $x = 3, y = 0$

Question 25.

If [3245]=[acbd][1001] write down the values of a,b,c and d

Solution:

Given

$$\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Comparing the corresponding elements

a = 3, b = 4, c = 2, d = 5

Question 26.

Find the value of x given that $A^2 = B$

Where A = [20121] and

B = [40x1]

Solution:

$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

$A^2 = B$

$$\Rightarrow A \times A = B$$

$$\Rightarrow \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\times2+12\times0 & 2\times12+12\times1 \\ 0\times2+1\times0 & 0\times12+1\times1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements of two equal matrices, $x = 36$

Question 27.

If A = [20x1] and B = [40361], find the value of x, given that A^2 – B

Solution:

Given

$$A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$$

$\because A^2 = B$

$$\therefore \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

Corresponding the corresponding elements

$3x = 36 \Rightarrow x = 12$

Hence $x = 12$

Question 28.

If A = $\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ find x and y when $A^2 = B$

Solution:

Given

A = $\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ find x and y when $A^2 = B$

Now, $A^2 = A \times A$

$$= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3x+x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

We have $A^2 = B$

Two matrices are equal if each and every corresponding element is equal.

Thus, $\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\Rightarrow 4x = 16$ and $1 = -y$

$\Rightarrow x = 4$ and $y = -1$

Question 29.

Find x, y if $\begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$

Solution:

Given

$$\begin{bmatrix}-2 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}-1\\ 2x\end{bmatrix}+3\begin{bmatrix}-2\\ 1\end{bmatrix}=2\begin{bmatrix}y\\ 3\end{bmatrix}$$

$$\begin{bmatrix}-2\times-1+0\times 2x\\ 3\times-1+1\times 2x\end{bmatrix}+\begin{bmatrix}-6\\ 3\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\begin{bmatrix}2\\ -3+2x\end{bmatrix}+\begin{bmatrix}-6\\ 3\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\begin{bmatrix}2-6\\ -3+2x+3\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}\Rightarrow\begin{bmatrix}-4\\ 2x\end{bmatrix}=\begin{bmatrix}2y\\ 6\end{bmatrix}$$

$$\Rightarrow 2x=6 \text{ and } 2y=-4$$

$$\Rightarrow x=\frac{6}{2} \text{ and } y=-\frac{4}{2}$$

$$\Rightarrow x=3 \text{ and } \Rightarrow y=-2$$

Question 30.

If $\begin{bmatrix}a & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}4 & 3\\ -3 & 2\end{bmatrix}=\begin{bmatrix}b & 11\\ 4 & c\end{bmatrix}$ find a,b and c

Solution:

$$\begin{bmatrix}a & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}4 & 3\\ -3 & 2\end{bmatrix}=\begin{bmatrix}b & 11\\ 4 & c\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}4a-3 & 3a+2\\ 4+0 & 3+0\end{bmatrix}=\begin{bmatrix}b & 11\\ 4 & c\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}4a-3 & 3a+2\\ 4 & 3\end{bmatrix}=\begin{bmatrix}b & 11\\ 4 & c\end{bmatrix}$$

Comparing the corresponding elements

$3a+2=11\Rightarrow 3a=11-2=9$

$$\therefore\ a=\frac{9}{3}=3$$

$4a-3=b\Rightarrow b=4\times 3-3=12-3=9$

$3=c$

Hence $a=3, b=9, c=3$

Question 31.
If A = [104−1] ,B = [20x−12] find the value of x if AB = BA
Solution:
Given

$$AB = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & x-2 \\ 0+0 & 0+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 8-x \\ 0+0 & 0+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{bmatrix}$$

$\because AB = BA$

$$\therefore \begin{bmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{bmatrix}$$

Comparing the corresponding elements

$$x - 2 = 8 - x \Rightarrow x + x = 8 + 2 \Rightarrow 2x = 10$$

$$\therefore \quad x = \frac{10}{2} = 5$$

Question 32.
If A = [2132] find x and y so that $A^2 - xA + yI$

Solution:

Given

A² = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \because A^2 = xA + yI$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = x\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2x & 3x \\ x & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$= \begin{bmatrix} 2x+y & 3x \\ x & 2x+y \end{bmatrix}$$

Comparing the corresponding elements

$3x = 12 \Rightarrow x = 4$

$2x + y = 7 \Rightarrow 2 \times 4 + y = 7$

$\Rightarrow 8 + y = 7 \Rightarrow y = 7 - 8 = -1$

Hence $x = 4, y = -1$

Question 33.

If P = [2369], Q = [3yx2]

find x and y such that PQ = 0

Solution:

Given

P = $\begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$,

Q = $\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$

$$PQ = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}$$

$\because PQ = 0$

$$\therefore \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the corresponding elements

$6 + 6y = 0 \Rightarrow 6y = -6 \quad \Rightarrow \quad y = -1$

$2x + 12 = 0 \Rightarrow 2x = -12 \Rightarrow x = -6$

Hence $x = -6, y = -1$

Question 34.
Let M×[1012]=[12] where M is a matrix
(i) State the order of matrix M
(ii) Find the matrix M
Solution:

Given

(i) M is the order of 1 x 2

let M = [x y]

$$\therefore \begin{bmatrix} x & y \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+0 & x+2y \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Comparing the corresponding elements

$x = 1$ and $x + 2y = 2 \Rightarrow 1 + 2y = 2$

$\Rightarrow 2y = 2 - 1 = 1 \Rightarrow y = \frac{1}{2}$

Hence $x = 1$, $y = \frac{1}{2}$ $\quad \therefore \mathrm{M} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$

Question 35.
Given [2−314] ,X = [76]
(i) the order of the matrix X
(ii) the matrix X

Solution:

We have

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}, X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

(*i*) We see that $2 \times 2 \times 2 \times 1 = 2 \times 1$

$\Rightarrow$ The order of X is 2×1

(*ii*) Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

So, $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$2x + y = 7$ ...(*i*)

$-3x + 4y = 6$ ...(*ii*)

Multiplying (*i*) by 3 and (*ii*) by 2, and adding we get :

$$\begin{aligned} 6x + 3y &= 21 \\ -6x + 8y &= 12 \\ \hline 11y &= 33 \quad \Rightarrow y = 3 \end{aligned}$$

From (*i*), $2x = 7 - 3 = 4 \Rightarrow x = 2$

So, $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Question 36.

Solve the matrix equation : [41] ,X = [−4−182]

Solution:

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix}, X = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$$

Let matrix X = [x y]

$$\therefore \begin{bmatrix} 4 \\ 1 \end{bmatrix}[x, y] = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 4x & 4y \\ x & y \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$$

Comparing the corresponding elements.

$4x = -4 \Rightarrow x = -1$

$4y = 8 \Rightarrow y = 2$

$\therefore X = [-1 \quad 2]$

Question 37.

(i) If A = $[2-4-15]$ and B = $[-32]$ find the matrix C such that AC = B

(ii) If A = $[2-4-15]$ and B = [0 -3] find the matrix C such that CA = B

Solution:

(i) given

$$A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Let matrix $C = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore\ AC = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - y \\ -4x + 5y \end{bmatrix}$$

But AC = B

$$\therefore \begin{bmatrix} 2x - y \\ -4x + 5y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Comparing the corresponding elements.

$2x - y = -3$ ...(*i*)

$-4x + 5y = 2$ ...(*ii*)

Multiplying (*i*) by 5 and (*ii*) by 1

$10x - 5y = -15$

$-4x + 5y = 2$

Adding, we get $6x = -13 \Rightarrow x = \dfrac{-13}{6}$

Substituting the value of x in (*i*)

$$2\left(\frac{-13}{6}\right) - y = -3 \Rightarrow \frac{-13}{3} - y = -3$$

$$-y = -3 + \frac{13}{3} = \frac{-9+13}{3} = \frac{4}{3}$$

$$\therefore\ y = -\frac{4}{3}$$

$$\therefore \text{ Matrix C} = \begin{bmatrix} \frac{-13}{6} \\ \frac{-4}{3} \end{bmatrix}$$

(*ii*) $A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ and $B = [0, -3]$

Let matrix $C = [x \quad y]$

Since the matrix A is 2×2 and $B = 7 \times 2$

$\because\ CA = B$

$\therefore\ (x\ \ y) \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} = [0\ \ -3]$

$= (2x - 4y \quad -x + 5y) = [0\ -3]$

Comparing,

$2x - 4y = 0 \quad \Rightarrow\ x - 2y = 0$

$\therefore \quad x = 2y$

and $-x + 5y = -3\ \Rightarrow\ -2y + 5y = -3$

$\Rightarrow\ 3y = -3 \quad \Rightarrow\ y = -1$

$\therefore\ x = 2y = 2 \times (-1) = -2$

Hence $C = [x \quad y] = [-2 \quad -1]$

Question 38.

If A = [3−1−42] , find matrix B such that BA = I,where I is unity matrix of order 2

Solution:

$A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

BA = I, where I is unity matrix of order 2

$\therefore\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore$ BA

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3a-b & -4a+2b \\ 3c-d & -4c+2d \end{bmatrix}$

$\therefore\ \begin{bmatrix} 3a-b & -4a+2b \\ 3c-d & -4c+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing the corresponding terms, we get

$3a - b = 1, -4a + 2b = 0 \Rightarrow 2b = 4a \Rightarrow \quad b = 2a$

$\therefore\ 3a - b = 1 \Rightarrow 3a - 2a = 1 \quad \Rightarrow \quad a = 1$

and $b = 2a \Rightarrow b = 2 \times 1 = 2$

$\therefore\ a = 1, b = 2$

and $3c - d = 0 \Rightarrow d = 3c$

$-4c + 2d = 1 \Rightarrow -4c + 2 \times 3c = 1$

$\Rightarrow -4c + 6c = 1 \Rightarrow 2c = 1 \Rightarrow c = \frac{1}{2}$

and $d = 3c = 3 \times \frac{1}{2} = \frac{3}{2}$

Hence $a = 1, b = 2, c = \frac{1}{2}, d = \frac{3}{2}$

$\therefore$ Matrix $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

Question 39.
If B = [−452−1] and C = [1747−1−13]
find the matrix A such that AB = C
Solution:

$B = \begin{bmatrix} -4 & 2 \\ 5 & -1 \end{bmatrix}$

$C = \begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$

and AB = C

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Then } AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} -4 & 2 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4a+5b & 2a-b \\ -4c+5d & 2c-d \end{bmatrix}$$

$\because AB = C$

$$\therefore \begin{bmatrix} -4a+5b & 2a-b \\ -4c+5d & 2c-d \end{bmatrix} = \begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$$

Comparing corresponding elements, we get

$\because -4a + 5b = 17$ ....(*i*)

$2a - b = -1$ ....(*ii*)

$-4c + 5d = 47$ ....(*iii*)

$2c - d = -13$ ....(*iv*)

Multiplying (*i*) by 1 and (*ii*) by 2

$\Rightarrow$ $-4a + 5b = 17$

$4a - 2b = -2$

Adding $\quad 3b = 15 \quad \Rightarrow b = \frac{15}{3} = 5$

$2a - b = -1 \Rightarrow 2a - 5 = -1 \quad \Rightarrow 2a = -1 + 5$

$= 4 \Rightarrow a = \frac{4}{2} = 2$

$\therefore\ a = 2, b = 5$

Again multiplying (*iii*) by 1 and (*iv*) by 2,

$$-4c + 5d = 47$$
$$4c - 2d = -26$$

Adding $\quad 3d = 21 \quad \Rightarrow d = \frac{21}{3} = 7$

and $2c - d = -13 \Rightarrow 2c - 7 = -13$

$\Rightarrow 2c = -13 + 7 = -6 \Rightarrow c = \frac{-6}{2} = -3$

$\therefore\ c = -3, d = 7$

Now matrix A = $\begin{bmatrix} 2 & 5 \\ -3 & 7 \end{bmatrix}$