Chapter 7 Ratio and Proportion Ex 7.3

Question 1.
If a : b : : c : d, prove that
(i) $\frac{2a+5b}{2a-5b}=\frac{2c+5d}{2c-5d}$
(ii) $\frac{5a+11b}{5c+11d}=\frac{5a-11b}{5c-11d}$
(iii) (2a + 3b)(2c – 3d) = (2a – 3b)(2c + 3d)
(iv) (la + mb) : (lc + mb) :: (la – mb) : (lc – mb)
Solution:

(i) a : b : : c : d

then $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{2a}{5b} = \frac{2c}{5d}$ (multiply by $\frac{2}{5}$)

Applying componendo and dividendo,

$$\frac{2a+5b}{2a-5b} = \frac{2c+5d}{2c-5d}$$

(ii) $\because a : b :: c : d$

$$\therefore \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{5a}{11b} = \frac{5c}{11d} \left(\text{Multiply by } \frac{5}{11}\right)$$

Applying componendo and dividendo,

$$\frac{5a+11b}{5a-11b} = \frac{5c+11d}{5c-11d}$$

$$\Rightarrow \frac{5a+11b}{5c+11d} = \frac{5a-11b}{5c-11d} \quad \text{(Applying alternendo)}$$

(*iii*) $\because a : b :: c : d$

$\therefore \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{2a}{3b} = \frac{2c}{3d}$ $\left(\text{Multiply by } \frac{2}{3}\right)$

Applying componendo and dividendo,

$$\frac{2a+3b}{2a-3b} = \frac{2c+3d}{2c-3d}$$

$\Rightarrow (2a + 3b)(2c - 3d)$

$= (2a - 3b)(2c + 3d)$ (By cross multiplication)

(*iv*) $\because a : b :: c : d$

$$\therefore \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{la}{mb} = \frac{lc}{md} \quad \left(\text{Multiply by } \frac{l}{m}\right)$$

Applying componendo and dividendo,

$$\frac{la+mb}{la-mb} = \frac{lc+md}{lc-md}$$

$$\Rightarrow \frac{la+mb}{lc+md} = \frac{la-mb}{lc-md} \quad \text{(By alternendo)}$$

$$\Rightarrow (la+mb) : (lc+md) :: (la-mb) : (lc-md)$$

Question 2.

(i) If $\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$, Show that $\frac{x}{y} = \frac{u}{v}$

(ii) $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, prove that $\frac{a}{b} = \frac{c}{d}$

Solution:

(i) $\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$

Applying alternendo $\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$

Applying componendo and dividendo

$$\frac{5x+7y+5x-7y}{5x+7y-5x+7y} = \frac{5u+7v+5u-7v}{5u+7u-5u+7v}$$

$$\Rightarrow \quad \frac{10x}{14y} = \frac{10u}{14v} \Rightarrow \frac{x}{y} = \frac{u}{v}$$

Hence proved. $\left(\text{Dividing by} = \frac{10}{14}\right)$

(ii) $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$

$\Rightarrow \frac{8a+5b}{8a-5b} = \frac{8c+5d}{8c-5d}$ (using alternendo)

Applying compoundo and dividendo,

$$\frac{8a+5b+8a-5b}{8a+5b-8a+5b} = \frac{8c+5d+8c-5c}{8c+5d-8c+5d}$$

$$\therefore \frac{16a}{10b} = \frac{16c}{10d}$$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$ (Dividing by $\frac{16}{10}$)

Hence proved.

Question 3.

If (4a + 5b) (4c – 5d) = (4a – 5d) (4c + 5d), prove that a, b, c, d are in proporton.

Solution:

(4a + 5b) (4c − 5d) = (4a − 5d) (4c + 5d)

$\Rightarrow \frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$

Applying componendo and dividendo

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$\Rightarrow \quad \frac{8a}{10b} = \frac{8c}{10d} \Rightarrow \frac{a}{b} = \frac{c}{d}$$

Hence, a, b, c, d are in proportion.

Question 4.

If (pa + qb) : (pc + qd) :: (pa – qb) : (pc – qd) prove that a : b : : c : d

Solution:

(pa + qb) : (pc + qd) :: (pa − qb) : (pc − qd)

$\Rightarrow \frac{pa+qb}{pc+qd} = \frac{pq-qb}{pc-qd}$

$\Rightarrow \frac{pa+qb}{pc-qd} = \frac{pq+qb}{pc-qd}$

Applying componendo and dividendo

$\Rightarrow \frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc-qd-pc+qd}$

$$\Rightarrow \frac{2\,pa}{2\,qb} = \frac{2\,pc}{2\,qd}$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Hence $a : b :: c = d$.

Question 5.

If (ma + nb): b :: (mc + nd) : d, prove that a, b, c, d are in proportion.

Solution:

(ma + nb): b :: (mc + nd) : d

$\Rightarrow \frac{ma+nb}{b} = \frac{mc+nd}{d}$

⇒ mad + nbd = mbc + nbd

⇒ mad = mbc

⇒ ad = bc

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Hence a : b :: c : d.

Question 6.

If $(11a^2 + 13b^2)(11c^2 – 13d^2) = (11a^2 – 13b^2)(11c^2 + 13d^2)$, prove that a : b :: c : d.

Solution:

$(11a^2 + 13b^2)(11c^2 - 13d^2) = (11a^2 - 13b^2)(11c^2 + 13d^2)$

$\Rightarrow \frac{11a+13b^2}{11a^2-13b^2} = \frac{11c^2+13d^2}{11c^2-13d^2}$

Applying componendo and dividendo

$$\frac{11a^2+13b^2+11a^2-13b^2}{11a^2+13b^2-11a^2+13b^2} = \frac{11c^2+13d^2+11c^2-13d^2}{11c^2+13d^2-11c^2+13d^2}$$

$$\Rightarrow \frac{22a^2}{26b^2} = \frac{22c^2}{26d^2}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{c^2}{d^2}$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Hence $a : b :: c : d$

Question 7.

If (a + 3b + 2c + 6d) (a – 3b – 2c + 6d) = (a + 3b – 2c – 6d) (a – 3b + 2c – 6d), prove that a : b :: c : d.

Solution:

$\frac{a+3b+2c+6d}{a-3b+2c-6d} = \frac{a+3b-2c-6d}{a-3b-2c+6d}$

$\Rightarrow \frac{a+3b+2c+6d}{a+3b-2c-6d} = \frac{a-3b+2c-6d}{a-3b-2c+6d}$ (by altenendo)

Applying componendo and dividendo

$$\frac{a+3b+2c+6d+a+3b-2c-6d}{a+3b+2c+6d-a-3b+2c+6d}$$

$$= \frac{a-3b+2c-6d+a-3b-2c+6d}{a-3b+2c-6d-a+3b+2c-6d}$$

$$\Rightarrow \frac{2(a+3b)}{2(2c+6d)} = \frac{2(a-3b)}{2(2c-6d)}$$

$$\Rightarrow \frac{a+3b}{2c+6d} = \frac{a-3b}{2c-6d} \qquad \text{(Dividing by 2)}$$

$$\Rightarrow \frac{a+3b}{a-3b} = \frac{2c+6d}{2c-6d} \qquad \text{(by alternendo)}$$

Again applying componendo and dividendo)

$$\frac{a+3b+a-3b}{a+3b-a+3b} = \frac{2c+6d+2c-6d}{2c+6d-2c+6d}$$

$$\Rightarrow \frac{2a}{6b} = \frac{4c}{12d} = \frac{2c}{6d}$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \qquad \left[\text{Dividing by } \frac{2}{6}\right]$$

Question 8.

If $x=\frac{2ab}{a+b}$ find the value of $\frac{x+a}{x-a}+\frac{x+b}{x-b}$

Solution:

$x = \frac{2ab}{a+b}$

$\Rightarrow \frac{x}{a} = \frac{2b}{a+b}$

Applying componendo and dividendo,

$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a} \qquad \text{...}(i)$$

Again $\frac{x}{b} = \frac{2a}{a+b}$

Applying componendo and dividendo,

$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \qquad \text{...}(ii)$$

Adding (*i*) and (*ii*)

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

$$= \frac{-a-3b+3a+b}{a-b}$$

$$= \frac{2a-2b}{a-b} = \frac{2(a-b)}{a-b} = 2$$

Question 9.

If $x = \frac{8ab}{a+b}$ find the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

Solution:

$x = \frac{8ab}{a+b}$

$\Rightarrow \frac{x}{4a} = \frac{2b}{a+b}$

Applying componendo and dividendo,

$$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a} \quad ...(i)$$

Again $\frac{x}{4b} = \frac{2a}{a+b}$

Applying compondndo and dividendo,

$$\frac{x+4b}{x-4b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \quad ...(ii)$$

Adding (i) and (ii)

$$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

$$= \frac{-a-3b+3a+b}{a-b} = \frac{2a-2b}{a-b}$$

$$= \frac{2(a-b)}{a-b} = 2$$

Question 10.

If x=46√2√+3√ find the value of x+22√x−22√+x+23√x−23√

Solution:

$x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$\Rightarrow \frac{4\sqrt{2}\times\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

$$\frac{x}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

Applying componendo and dividendo,

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad \ldots(i)$$

Again $\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$

Applying componendo and dividendo,

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad \ldots(ii)$$

Adding (*i*) and (*ii*),

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}} = 2$$

Question 11.
Solve x:36x+1√+6x√36x+1√−6x√=9

Solution:

$\frac{\sqrt{36x+1}+6\sqrt{x}}{\sqrt{36x+1}-6\sqrt{x}}=\frac{9}{1}$

Applying componendo and dividendo,

$$\frac{\sqrt{36x+1}+6\sqrt{x}+\sqrt{36x+1}-6\sqrt{x}}{\sqrt{36x+1}+6\sqrt{x}-\sqrt{36x-1}+6\sqrt{x}}$$

$$=\frac{9+1}{9-1} \Rightarrow \frac{2\sqrt{36x+1}}{12\sqrt{x}}=\frac{10}{8}$$

$$\Rightarrow \frac{\sqrt{36x+1}}{6\sqrt{x}}=\frac{5}{4} \text{ (Squaring both sides)}$$

$$\frac{36x+1}{36x}=\frac{25}{16}$$

$$\Rightarrow \quad 36x \times 25 = 16(36x+1)$$

$$\Rightarrow 900x = 576x + 16 \Rightarrow 900x - 576x = 16$$

$$\Rightarrow \quad 324x = 16$$

$$\therefore \quad x = \frac{16}{324} = \frac{4}{81}$$

Question 12.
Find x from the following equations :
(i) 2−x√+2+x√2−x√−2+x√=3
(ii) x+4√+x−10√x+4√−x−10√=52
(iii) 1+x√+1−x√1+x√−1−x√=ab
(iv) 12x+1√+2x−3√12x+1√−2x−3√=32
(v) 3x+9x2−5√3x−9x2−5√=5
(vi) a+x√+a−x√a+x√−a−x√=cd
Solution:

(i) $\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}=3$

Applying componendo and dividendo,

$$\frac{\sqrt{2-x}+\sqrt{2+x}+\sqrt{2-x}-\sqrt{2+x}}{\sqrt{2-x}+\sqrt{2+x}-\sqrt{2-x}+\sqrt{2+x}}=\frac{3+1}{3-1}$$

$$\Rightarrow \quad \frac{2\sqrt{2-x}}{2\sqrt{2+x}}=\frac{4}{2} \Rightarrow \frac{\sqrt{2-x}}{\sqrt{2+x}}=\frac{2}{1}$$

Squaring both sides

$$\frac{2-x}{2+x}=\frac{4}{1} \Rightarrow 8+4x=2-x$$

$$4x+x=2-8 \Rightarrow 5x=-6$$

$$\therefore \ x=\frac{-6}{5}$$

(*ii*) $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+4}+\sqrt{x-10}+\sqrt{x+4}-\sqrt{x-10}}{\sqrt{x+4}+\sqrt{x-10}-\sqrt{x+4}+\sqrt{x-10}}=\frac{5+2}{5-2}$$

$$\Rightarrow \quad \frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{7}{3} \Rightarrow \frac{\sqrt{x+4}}{\sqrt{x-10}}=\frac{7}{3}$$

Squaring both sides

$$\frac{2-x}{2+x} = \frac{4}{1} \Rightarrow 8 + 4x = 2 - x$$

$$4x + x = 2 - 8 \Rightarrow 5x = -6$$

$$\therefore\ x = \frac{-6}{5} \textbf{ Ans.}$$

(*ii*) $\dfrac{\sqrt{x+4} + \sqrt{x-10}}{\sqrt{x+4} - \sqrt{x-10}} = \dfrac{5}{2}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+4} + \sqrt{x-10} + \sqrt{x+4} - \sqrt{x-10}}{\sqrt{x+4} + \sqrt{x-10} - \sqrt{x+4} + \sqrt{x-10}} = \frac{5+2}{5-2}$$

$$\Rightarrow \quad \frac{2\sqrt{x+4}}{2\sqrt{x-10}} = \frac{7}{3} \Rightarrow \frac{\sqrt{x+4}}{\sqrt{x-10}} = \frac{7}{3}$$

Squaring both sides,

$$\frac{x+4}{x-10} = \frac{49}{9} \Rightarrow 49x - 490 = 9x + 36$$

$$\Rightarrow 49x - 9x = 36 + 490 \Rightarrow \quad 40x = 526$$

$$\therefore\ x = \frac{526}{40} = \frac{263}{20}$$

(iii) $\dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=\dfrac{a}{b}$

Applying componendo and dividendo,

$$\frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}}=\frac{a+b}{a-b}$$

$$\Rightarrow \frac{2\sqrt{1+x}}{2\sqrt{1-x}}=\frac{a+b}{a-b} \quad \Rightarrow \frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{a+b}{a-b}$$

Squaring both sides, $\dfrac{1+x}{1-x}=\dfrac{(a+b)^2}{(a-b)^2}$

Again applying componendo and dividendo,

$$\frac{1+x+1-x}{1+x-1+x}=\frac{(a+b)^2+(a-b)^2}{(a+b)^2-(a-b)^2}$$

$$\Rightarrow \frac{2}{2x}=\frac{2(a^2+b^2)}{4ab} \quad \Rightarrow \quad \frac{1}{x}=\frac{a^2+b^2}{2ab}$$

$$\therefore\ x=\frac{2ab}{a^2+b^2}$$

(iv) $\dfrac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}}=\dfrac{3}{2}$

Applying componendo and dividendo,

$$\frac{\sqrt{12x+1}+\sqrt{2x-3}+\sqrt{12x+1}-\sqrt{2x-3}}{\sqrt{12x+1}+\sqrt{2x-3}-\sqrt{12x+1}+\sqrt{2x-3}}$$

$$=\frac{3+2}{3-2}$$

$$\Rightarrow \frac{2\sqrt{12x+1}}{2\sqrt{2x-3}}=\frac{5}{1}$$

$$\Rightarrow \frac{\sqrt{12x+1}}{\sqrt{2x-3}}=\frac{5}{1}$$

Squaring both sides, $\dfrac{12x+1}{2x-3}=\dfrac{25}{1}$

$\Rightarrow \quad 50x-75=12x+1$

$\Rightarrow \quad 50x-12x=1+75$

$\Rightarrow \quad 38x=76 \Rightarrow x=\dfrac{76}{38}=2$

$\therefore x=2$

(v) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}} = \frac{5}{1}$

Applying componendo and dividendo,

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}} = \frac{5+1}{5-1}$$

$$\frac{6x}{2\sqrt{9x^2-5}} = \frac{6}{4} \Rightarrow \frac{3x}{\sqrt{9x^2-5}} = \frac{3}{2}$$

Squaring both sides

$$\frac{9x^2}{9x^2-5} = \frac{9}{4} \Rightarrow 81\,x^2 - 45 = 36\,x^2$$

$\Rightarrow \quad 81\,x^2 - 36\,x^2 = 45 \Rightarrow 45\,x^2 = 45$

$\Rightarrow \quad x^2 = 1 \Rightarrow x = \pm 1$

$\therefore x = 1, -1$

Check : (*i*) When $x = 1$,

then in the given equation

$$\frac{3\times 1+\sqrt{9\times 1-5}}{3\times 1-\sqrt{9\times 1-5}} = \frac{3+\sqrt{4}}{3-\sqrt{4}} = \frac{3+2}{3-2} = \frac{5}{1}$$

which is given

$\therefore x = 1$

(*ii*) When $x = -1$, then

$$\frac{3(-1)+\sqrt{9(-1)^2-5}}{3(-1)-\sqrt{9(-1)^2-5}}$$

$$= \frac{-3+\sqrt{9-5}}{-3-\sqrt{9-5}} = \frac{-3+\sqrt{4}}{-3-\sqrt{4}}$$

$$= \frac{-3+2}{-3-2} = \frac{-1}{-5} = \frac{1}{5} \neq \frac{5}{1}$$

$\therefore x = -1$ is not its solution.

Hence $x = 1$

(*vi*) $$\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{c}{d}$$

Applying componendo and dividendo,

$$\frac{\sqrt{a+x}+\sqrt{a-x}+\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}-\sqrt{a+x}+\sqrt{a-x}} = \frac{c+d}{c-d}$$

$$\Rightarrow \frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{c+d}{c-d} \Rightarrow \frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{c+d}{c-d}$$

Squaring both sides $\frac{a+x}{a-x} = \frac{(c+d)^2}{(c-d)^2}$

Again applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(c+d)^2+(c-d)^2}{(c+d)^2-(c-d)^2}$$

$$\Rightarrow \frac{2a}{2x} = \frac{2(c^2+d^2)}{4cd} \Rightarrow \frac{a}{x} = \frac{c^2+d^2}{2cd}$$

$$\Rightarrow \quad x(c^2+d^2) = 2acd$$

$$\Rightarrow \quad x = \frac{2acd}{c^2+d^2}$$

Question 13.

Solve 1+x+x2 1−x+x2 =62(1+x)63(1−x)

Solution:

$$\frac{1+x+x^2}{1-x+x^2} = \frac{62(1+x)}{63(1-x)}$$

$$\Rightarrow \frac{(1-x)\left(1+x+x^2\right)}{(1+x)(1-x+x^2)} = \frac{62}{63}$$

$$\Rightarrow \frac{(1+x)\left(1-x+x^2\right)}{(1-x)(1+x+x^2)} = \frac{63}{62}$$

$$\Rightarrow \quad \frac{1+x^3}{1-x^3} = \frac{63}{62}$$

Applying componendo and dividendo,

$$\frac{1+x^3+1-x^3}{1+x^3-1+x^3} = \frac{63+62}{63-62}$$

$$\Rightarrow \frac{2}{2x^3} = \frac{125}{1} \Rightarrow \quad \frac{1}{x^3} = \frac{125}{1}$$

$$\Rightarrow \quad x^3 = \frac{1}{125} = \left(\frac{1}{5}\right)^3$$

$$\therefore \quad x = \frac{1}{5}$$

Question 14.

Solve for x:16(a−xa+x)3=a+xa−x

Solution:

$x : 16\left(\frac{a-x}{a+x}\right)^3 = \frac{a+x}{a-x}$

$\Rightarrow \left(\frac{a+x}{a-x}\right) \times \left(\frac{a+x}{a-x}\right)^3 = 16$

$$\Rightarrow \left(\frac{a+x}{a-x}\right)^4 = 16 = (\pm 2)^4 \Rightarrow \quad \frac{a+x}{a-x} = \pm 2$$

When $\frac{a+x}{a-x} = \frac{2}{1}$

Applying componendo and dividendo,

$$\frac{a+x+a-x}{a+x-a+x} = \frac{2+1}{2-1} \Rightarrow \frac{2a}{2x} = \frac{3}{1} \Rightarrow \frac{a}{x} = \frac{3}{1}$$

$\Rightarrow 3x = a \qquad \therefore\ x = \frac{a}{3}$

when $\frac{a+x}{a-x} = \frac{-2}{1}$

Applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{-2+1}{-2-1} \Rightarrow = \frac{2a}{2x} = \frac{-1}{-3}$$

$\Rightarrow \frac{a}{x} = \frac{1}{3} \Rightarrow x = 3a$

Hence $x = \frac{a}{3}, 3a$

Question 15.

If x=a+x√+a−1√a+1√−a−1√ , using properties of proportion , show that $x^2 - 2ax + 1 = 0$

Solution:

We have $x = \frac{\sqrt{a+x}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$

(Applying componendo and dividendo)

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$

$$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{2a}{2}$$

(Again applying componendo and dividendo)

$$\Rightarrow \frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = a$$

$$\Rightarrow \frac{2x^2+2}{4x} = a \quad \Rightarrow \frac{2(x^2+1)}{4x} = a$$

$$\Rightarrow \frac{(x^2+1)}{2x} = a \quad \Rightarrow 2ax = x^2+1$$

$$\Rightarrow x^2 - 2ax + 1 = 0$$

Proved.

Question 16.

Given $x=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$ Use componendo and dividendo to prove that $b^2=\frac{2a^2x}{x^2+1}$

Solution:

If $\frac{x}{1}=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$

Applying componendo and dividendo both sides

$$\frac{x+1}{x-1}=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

$$\Rightarrow\frac{x+1}{x-1}=\frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2+b^2}}\Rightarrow\frac{x+1}{x-1}=\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

Squaring, both sides we have

$$\Rightarrow\frac{(x+1)^2}{(x-1)^2}=\frac{a^2+b^2}{a^2-b^2}\Rightarrow\frac{x^2+1+2x}{x^2+1-2x}=\frac{a^2+b^2}{a^2-b^2}$$

Applying componendo and dividendo both sides

$$\Rightarrow\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x}=\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

$$\frac{2x^2+2}{4x}=\frac{2a^2}{2b^2}\Rightarrow\frac{x^2+1}{2x}=\frac{a^2}{b^2}\Rightarrow b^2=\frac{2a^2x}{x^2+1}$$

Question 17.

Given that $\frac{a^3+3ab^2}{b^3+3a^2b}=\frac{63}{62}$. Using componendo and dividendo find a: b. (2009)

Solution:

Given that $\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$

By componendo and dividendo

$$\frac{a^3+3ab^2+b^3+3a^2b}{a^3+3ab^2-b^3-3a^2b} = \frac{63+62}{63-62} = \frac{125}{1}$$

$$\Rightarrow \frac{(a+b)^3}{(a-b)^3} = \left(\frac{5}{1}\right)^3 \Rightarrow \frac{a+b}{a-b} = 5 \Rightarrow a+b = 5a-5b$$

$$\Rightarrow 5a - a - 5b - b = 0 \Rightarrow 4a - 6b = 0 \Rightarrow 4a = 6b$$

$$\Rightarrow \frac{a}{b} = \frac{6}{4} \Rightarrow \quad \frac{a}{b} = \frac{3}{2}$$

a : b = 3 : 2

Question 18.

Give $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$ Using componendo and dividendo find x : y.

Solution:

Give $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$

Using componendo-dividendo, we have

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(9y-3)^3}$$

$$\Rightarrow \left(\frac{x+2}{x-2}\right)^3 = \left(\frac{y+3}{y-3}\right)^3$$

$$\Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Again using componendo-dividendo, we get

$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

$$\Rightarrow \frac{2x}{4} = \frac{2y}{3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{3}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{3}$$

Thus the required ratio is $x : y = 2 : 3$

Question 19.

Using the properties of proportion, solve the following equation for x; given

x3+3x3x2+1=34191

Solution:

$$\frac{x^3+3x}{3x^2+1} = \frac{341}{91}$$

Applying componendo and dividendo

$$\frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1} = \frac{341+91}{341-91}$$

$$\Rightarrow \frac{x^3+3x^2+3x+1}{x^3-3x^2+3x-1} = \frac{432}{250} = \frac{216}{125}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{216}{125} = \left(\frac{6}{5}\right)^3$$

$$\therefore \frac{x+1}{x-1} = \frac{6}{5} \Rightarrow 6x-6 = 5x+5$$

$$\Rightarrow \quad 6x-5x = 5+6 \Rightarrow x = 11$$

Question 20.

If $\frac{x+y}{ax+by}=\frac{y+z}{ay+bz}=\frac{z+x}{az+bx}$, prove that each of these ratio is equal to $\frac{2}{a+b}$ unless x + y + z = 0

Solution:

$$\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$$

$$= \frac{x+y+y+z+z+x}{ax+by+ay+bz+az+bx}$$

$$= \frac{2(x+y+z)}{x(a+b)+y(a+b)+z(a+b)}$$

$$= \frac{2(x+y+z)}{(a+b)(x+y+z)} = \frac{2}{a+b} \text{ if } x+y+z \neq 0.$$

Hence proved.