Chapter 6 Factorization Ex 6

Question 1.
Find the remainder (without divisions) on dividing f(x) by x – 2, where
(i) $f(x) = 5x^2 – 1x + 4$
(ii) $f(x) = 2x^3 – 7x^2 + 3$
Solution:

Let $x - 2 = 0$, then $x = 2$

(i) Substituting value of x in f(x)

$f(x) = 5x^2 - 7x + 4$

$\Rightarrow f(2) = 5(2)^2 - 7(2) + 4$

$\Rightarrow f(2) = 20 - 14 + 4 = 10$

Hence Remainder = 10

(ii) $f(x) = 2x^3 - 7x^2 + 3$

$\therefore f(2) = 2(2)^3 - 7(2)^2 + 3 = 16 - 28 + 3$

Hence Remainder = – 9

Question 2.
Using remainder theorem, find the remainder on dividing f(x) by (x + 3) where
(i) $f(x) = 2x^2 – 5x + 1$
(ii) $f(x) = 3x^3 + 7x^2 – 5x + 1$
Solution:

Let $x + 3 = 0$

$\Rightarrow x = -3$

Substituting the value of x in f(x),

(i) $f(x) = 2x^2 - 5x + 1$

$\therefore f(-3) = 2(-3)^2 - 5(-3) + 1$

$= 18 + 15 + 1 = 34$

Hence Remainder = 34

(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

$= 3(-3)^3 + 7(-3)^2 - 5(-3) + 1$

$= -81 + 63 + 15 + 1 = -2$

Hence Remainder = –2

Question 3.
Find the remainder (without division) on dividing f(x) by (2x + 1) where
(i) $f(x) = 4x^2 + 5x + 3$
(ii) $f(x) = 3x^3 – 7x^2 + 4x + 11$

Solution:

Let 2x + 1 = 0, then x = $-\frac{1}{2}$

Substituting the value of x in f(x):

(i) $f(x) = 4x^2 + 5x + 3$

$$= 4\left(-\frac{1}{2}\right)^2 + 5 \times \left(-\frac{1}{2}\right) + 3$$

$$= 4 \times \frac{1}{4} - \frac{5}{2} + 3 = 1 - \frac{5}{2} + 3 = 4 - \frac{5}{2} = \frac{3}{2}$$

$$\therefore \text{ Remainder } = \frac{3}{2}$$

(ii) $f(x) = 3x^3 - 7x^2 + 4x + 11$

$$= -3\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 11$$

$$= 3\left(-\frac{1}{8}\right) - 7\left(\frac{1}{4}\right) + 4\left(-\frac{1}{2}\right) + 11$$

$$= -\frac{3}{8} - \frac{7}{4} - 2 + 11$$

$$= \frac{-3 - 14 - 16 + 88}{8} = \frac{55}{8} = 6\frac{7}{8}$$

$$\therefore \text{ Remainder } = 6\frac{7}{8}$$

Question 4.

(i) Find the remainder (without division) when $2x^3 - 3x^2 + 7x - 8$ is divided by x – 1 (2000)

(ii) Find the remainder (without division) on dividing $3x^2 + 5x - 9$ by (3x + 2)

Solution:

(i) Let x − 1 = 0, then x = 1

Substituting value of x in f(x)

$$f(x) = 2x^3 - 3x^2 + 7x - 8$$
$$= 2(1)^3 - 3(1)^2 + 7(1) - 8$$
$$= 2 \times 1 - 3 \times 1 + 7 \times 1 - 8 = 2 - 3 + 7 - 8$$
$$= -2$$

$\therefore$ Remainder = 2

(ii) Let $3x + 2 = 0$, then $3x = -2 \Rightarrow x = \frac{-2}{3}$

Substituting the value of x in $f(x)$

$$f(x) = 3x^2 + 5x - 9$$
$$= 3\left(-\frac{2}{3}\right)^2 + 5\left(-\frac{2}{3}\right) - 9$$
$$= 3 \times \frac{4}{9} - 5 \times \frac{2}{3} - 9 = \frac{4}{3} - \frac{10}{3} - 9$$
$$= -\frac{6}{3} - 9 = -2 - 9 = -11$$

$\therefore$ Remainder = −11

Question 5.

Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by x – 2, leaves a remainder 7. (2016)

Solution:

$f(x) = 2x^2 + 3x^2 - kx + 5$

$g(x) = x - 2$, if $x - 2 = 0$, then $x = 2$

Dividing f(x) by g(x) the remainder will be

$$f(2) = 2(2)^3 + 3(2)^2 - k \times 2 + 5$$
$$= 16 + 12 - 2k + 5 = 33 - 2k$$

Remainder = 7

$$\therefore 33 - 2k = 7 \Rightarrow 33 - 7 = 2k$$
$$\Rightarrow 2k = 26 \Rightarrow k = \frac{26}{2} = 13$$
$$\therefore k = 13$$

Question 6.

Using remainder theorem, find the value of a, if the division of $x^3 + 5x^2 - ax + 6$

by (x – 1) leaves the remainder 2a.
Solution:

Let x − 1 = 0, then x = 1

Substituting the value of x in f(x)

$$f(x) = x^3 + 5x^2 - ax + 6$$
$$= (1)^3 + 5(1)^2 - a(1) + 6 = 1 + 5 - a + 6 = 12 - a$$
$$\because \text{Remainder} = 2a$$
$$\therefore 12 - a = 2a$$
$$\Rightarrow 12 = a + 2a \Rightarrow 3a = 12$$
$$\therefore a = 4$$

Question 7.
(i) What number must be subtracted from $2x^2 - 5x$ so that the resulting polynomial leaves the remainder 2 when divided by 2x + 1?
(ii) What number must be added to $2x^3 - 7x^2 + 2x$ so that the resulting polynomial leaves the remainder -2 when divided by 2x – 3?
Solution:

(i) Let a be subtracted from $2x^2 - 5x$,

Dividing $2x^2 - 5x$ by $2x + 1$,

$$
\begin{array}{r|l}
2x+1 & 2x^2 - 5x - a \; (x - 3 \\
 & \underline{2x^2 + x} \\
 & -\quad - \\
 & \quad -6x - a \\
 & \quad \underline{-6x - 3} \\
 & \quad +\quad + \\
 & \quad \underline{-a + 3}
\end{array}
$$

Here remainder is $(3 - a)$

but we are given that remainder is 2.

$\therefore\ 3 - a = 2$

$\Rightarrow -a = 2 - 3 = -1 \Rightarrow a = 1$

Hence 1 is to be subtracted.

(*ii*) Let a be added to $2x^3 - 7x^2 + 2x$ dividing it by $2x - 3$, then

$$
\begin{array}{r|l}
2x-3 & 2x^3 - 7x^2 + 2x + a \; (x^2 - 2x - 2 \\
 & 2x^3 - 3x^2 \\
 & \underline{-\quad +} \\
 & \quad -4x^2 + 2x \\
 & \quad -4x^2 + 6x \\
 & \quad \underline{+\quad -} \\
 & \qquad -4x + a \\
 & \qquad -4x + 6 \\
 & \qquad \underline{+\quad -} \\
 & \qquad \underline{a - 6}
\end{array}
$$

But remainder is -2, then

$a - 6 = -2 \Rightarrow a = -2 + 6 \Rightarrow a = 4$

Hence 4 is to be added.

Question 8.

(i) When divided by $x - 3$ the polynomials $x^2 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'

(ii) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by $x + 3$.

Solution:

By dividing

$x^3 - px^2 + x + 6$

and $2x^3 - x^2 - (p + 3)x - 6$

by $x - 3$, the remainder is same

Let $x - 3 = 0$, then $x = 3$

Now by Remainder Theorem,

Let $p(x) = x^3 - px^2 + x + 6$

$p(3) = (3)^3 - p(3)^2 + 3 + 6$

$= 27 - 9p + 9 = 36 - 9p$

and $q(x) = 2x^3 - x^2 - (p + 3)x - 6$

$q(3) = 2(3)^2 - (3)^2 - (p + 3) \times 3 - 6$

$= 2 \times 27 - 9 - 3p - 9 - 6$

$= 54 - 24 - 3p = 30 - 3p$

$\because$ The remainder in each case is same

$\therefore$ $36 - 9p = 30 - 3p$

$$36 - 30 = 9p - 3p \Rightarrow 6 = 6p \Rightarrow p = \frac{6}{6} = 1$$

$\therefore$ $p = 1$

(*ii*) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by $x + 3$.

The given polynomials are $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$

Let $p(x) = ax^3 + 3x^2 - 9$

and $q(x) = 2x^3 + 4x + a$

Given that $p(x)$ and $q(x)$ leave the same remainder when divided by $(x + 3)$,

Thus by Remainder Theorem, we have

$p(-3) = q(-3)$

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$

$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$

$\Rightarrow -27a + 18 = -66 + a$

$\Rightarrow -27a - a = -66 - 18 \Rightarrow -28a = -84$

$$\Rightarrow a = \frac{84}{28}$$

$\therefore$ $a = 3$

Question 9.
By factor theorem, show that (x + 3) and (2x – 1) are factors of $2x^2 + 5x - 3$.
Solution:

Let x + 3 = 0 then x = − 3

Substituting the value of x in f(x)

$f(x) = 2x^2 + 5x - 3 = 2(-3)^2 + 5(-3) - 3$
$f(-3) = 18 - 15 - 3 = 0$

$\because$ Remainder = 0, then $x + 3$ is a factor

Again let $2x - 1 = 0$, then $x = \frac{1}{2}$

Substituting the value of x in $f(x)$,

$f(x) = 2x^2 + 5x - 3$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 3$$

$$= 2 \times \frac{1}{4} + \frac{5}{2} - 3 = \frac{1}{2} + \frac{5}{2} - 3 = 0$$

$\because$ Remainder = 0,

$\therefore$ $2x - 1$ is also a factor Hence proved.

Question 10.
Show that (x – 2) is a factor of $3x^2 - x - 10$. Hence factorise $3x^2 - x - 10$.
Solution:

Let x − 2 = 0, then x = 2

Substituting the value of x in f(x),

$f(x) = 3x^2 - x - 10 = 3(2)^2 - 2 - 10 = 12 - 2 - 10 = 0$

$\because$ Remainder is zero

$\therefore$ $x - 2$ is a factor of $f(x)$.

Dividing $3x^2 - x - 10$ by $x - 2$, we get

$$\begin{array}{r|l} x-2 & 3x^2 - x - 10 \;(3x+5 \\ & \underline{3x^2 - 6x} \\ & \underline{-\quad +} \\ & \quad 5x - 10 \\ & \quad 5x - 10 \\ & \underline{\quad - \quad +} \\ & \underline{\quad\quad \times} \end{array}$$

$\therefore$ $3x^2 - x - 10 = (x - 2)(3x + 5)$

Question 11.
Show that (x – 1) is a factor of $x^3 - 5x^2 - x + 5$ Hence factorise $x^3 - 5x^2 - x + 5$.

Solution:

Let x − 1 = 0, then x = 1

Substituting the value of x in f(x),

$f(x) = x^3 - 5x^2 - x + 5$

$= (1)^3 - 5(1)^2 - 1 + 5 = 1 - 5 - 1 + 5 = 0$

$\because$ Remainder = 0

$\therefore x - 1$ is a factor of $x^3 - 5x^2 - x + 5$

Now dividing $f(x)$ by $x - 1$, we get

$$
\begin{array}{r|l}
x - 1 & x^3 - 5x^2 - x + 5 \;(x^2 - 4x - 5 \\
 & x^3 - x^2 \\
 & \underline{- \quad +} \\
 & -4x^2 - x \\
 & -4x^2 + 4x \\
 & \underline{+ \qquad -} \\
 & -5x + 5 \\
 & -5x + 5 \\
 & \underline{+ \quad -} \\
 & \times
\end{array}
$$

$\therefore x^3 - 5x^2 - x + 5$

$= (x - 1)(x^2 - 4x - 5) = (x - 1)[x^2 - 5x + x - 5]$

$= (x - 1)[x(x - 5) + 1(x - 5)]$

$= (x - 1)(x + 1)(x - 5)$

Question 12.

Show that (x – 3) is a factor of $x^3 - 7x^2 + 15x - 9$. Hence factorise $x^3 - 7x^2 + 15x - 9$

Solution:

Let x − 3 = 0, then x = 3,

Substituting the value of x in f(x),

$f(x) = x^3 - 7x^2 + 15x - 9 = (3)^3 - 7(3)^2 + 15(3) - 9$

$= 27 - 63 + 45 - 9 = 72 - 72 = 0$

$\because$ Remainder = 0,

$\therefore x - 3$ is a factor of $x^3 - 7x^2 + 15x - 9$

Now dividing it by $x - 3$, we get

$$
\begin{array}{r|l}
x-3 & x^3 - 7x^2 + 15x - 9 \; (x^2 - 4x + 3 \\
 & x^3 - 3x^2 \\
 & -\quad + \\
\hline
 & -4x^2 + 15x \\
 & -4x^2 + 12x \\
 & +\quad - \\
\hline
 & 3x - 9 \\
 & 3x - 9 \\
 & -\quad + \\
\hline
 & \times \\
\hline
\end{array}
$$

$\therefore x^3 - 7x^2 + 15x - 9$

$= (x - 3)(x^2 - 4x + 3) = (x - 3)[x^2 - x - 3x + 3]$

$= (x - 3)[x(x - 1) - 3(x - 1)]$

$= (x - 3)(x - 1)(x - 3) = (x - 3)^2 (x - 1)$

Question 13.

Show that (2x + 1) is a factor of $4x^3 + 12x^2 + 11x + 3$. Hence factorise $4x^3 + 12x^2 + 11x + 3$.

Solution:

Let 2x + 1 = 0,

then x = $-\frac{1}{2}$

Substituting the value of x in f(x),

$$f(x) = 4x^3 + 12x^2 + 11x + 3$$

$$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 + 11\left(-\frac{1}{2}\right) + 3$$

$$= 4\left(-\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) + 11\left(-\frac{1}{2}\right) + 3$$

$$= -\frac{1}{2} + 3 - \frac{11}{2} + 3 = (6) - (6) = 0$$

$\because$ Remainder = 0

$\therefore$ $2x + 1$ is a factor of $4x^3 + 12x^2 + 11x + 3$

Now dividing $f(x)$ by $2x + 1$, we get

$$
\begin{array}{l}
2x+1\overline{)4x^3+12x^2+11x+3}(2x^2+5x+3 \\
\quad\quad\;\; 4x^3+2x^2 \\
\quad\quad\;\; \underline{-\quad -} \\
\quad\quad\quad\quad 10x^2+11x \\
\quad\quad\quad\quad 10x^2+5x \\
\quad\quad\quad\quad \underline{-\quad\; -} \\
\quad\quad\quad\quad\quad\quad 6x+3 \\
\quad\quad\quad\quad\quad\quad 6x+3 \\
\quad\quad\quad\quad\quad\quad \underline{-\quad -} \\
\quad\quad\quad\quad\quad\quad\quad \times
\end{array}
$$

$\therefore$ $4x^3 + 12x^2 + 11x + 3$

$= (2x + 1)(2x^2 + 5x + 3)$

$= (2x + 1)[2x^2 + 2x + 3x + 3]$

$= (2x + 1)[2x(x + 1) + 3(x + 1)]$

$= (2x + 1)[(x + 1)(2x + 3)]$

$= (2x + 1)(x + 1)(2x + 3)$

Question 14.
Show that 2x + 7 is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence factorize the given expression completely, using the factor theorem. (2006)
Solution:

Let 2x + 7 = 0, then 2x = -7

$x = \frac{-7}{2}$

substituting the value of x in f(x),

$$f(x) = 2x^3 + 5x^2 - 11x - 14$$

$$f\left(-\frac{7}{2}\right) = 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14$$

$$= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14$$

$$= \frac{-343+245+154-56}{4} = \frac{-399+399}{4} = 0$$

Hence, $(2x + 7)$ is a factor of $f(x)$.
Proved.

Now, $2x^3 + 5x^2 - 11x - 14 = (2x + 7)(x^2 - x - 2)$

$= (2x + 7)[x^2 - 2x + x - 2]$

$= (2x + 7)[x(x - 2) + 1(x - 2)]$

$= (2x + 7)(x + 1)(x - 2)$ Ans.

$$\begin{array}{r|l} 2x+7 & 2x^3 + 5x^2 - 11x - 14 \;(x^2 - x - 2 \\ & 2x^3 + 7x^2 \\ & - \quad - \\ \hline & -2x^2 - 11x \\ & -2x^2 - 7x \\ & + \quad + \\ \hline & -4x - 14 \\ & -4x - 14 \\ & + \quad + \\ \hline & \times \end{array}$$

Question 15.
Use factor theorem to factorise the following polynominals completely.

(i) $x^3 + 2x^2 - 5x - 6$
(ii) $x^3 - 13x - 12$.
Solution:

(i) Let $f(x) = x^3 + 2x^2 - 5x - 6$

Factors of $(\because 6 = \pm 1 ; \pm 2, \pm 3, \pm 6)$

Let $x = -1$, then

$f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$

$= -1 + 2(1) + 5 - 6 = -1 + 2 + 5 - 6 = 7 - 7 = 0$

$\because f(-1) = 0$

$\therefore x + 1$ is a factor of $f(x)$

Now, dividing $f(x)$ by $x + 1$, we get

$f(x) = (x + 1)(x^2 + x - 6)$

$= (x + 1)(x^2 + 3x - 2x - 6)$

$= (x + 1)\{x(x + 3) - 2(x + 3)\}$

$= (x + 1)(x + 3)(x - 2)$

$$
\begin{array}{l}
x+1\,\big)\,x^3 + 2x^2 - 5x - 6\,\big(\,x^2 + x - 6 \\
\qquad\;\; x^3 + x^2 \\
\qquad\;\; - \quad - \\
\hline
\qquad\qquad x^2 - 5x \\
\qquad\qquad x^2 + x \\
\qquad\qquad - \quad - \\
\hline
\qquad\qquad\quad -6x - 6 \\
\qquad\qquad\quad -6x - 6 \\
\qquad\qquad\quad + \quad + \\
\hline
\qquad\qquad\qquad \times \\
\hline
\end{array}
$$

(*ii*) $f(x) = x^3 - 13x - 12$

Let $x = 4$, then

$f(x) = (4)^3 - 13(4) - 12$

$= 64 - 52 - 12 = 64 - 64 = 0$

$\because f(x) = 0$

$\therefore x - 4$ is a factor of $f(x)$

Now, dividing $f(x)$ by $(x - 4)$, we get,

$f(x) = (x - 4)(x^2 + 4x + 3)$

$= (x - 4)(x^2 + 3x + x + 3)$

$= (x - 4)[x(x + 3) + 1(x + 3)]$

$= (x - 4)(x + 3)(x + 1)$ **Ans.**

$x - 4\,)\,x^3 - 13x - 12\,(\,x^2 + 4x + 3$

$x^3 - 4x^2$

$-\quad +$

$4x^2 - 13x$

$4x^2 - 16x$

$-\quad +$

$3x - 12$

$3x - 12$

$-\quad +$

$\times$

Question 16.

(i) Use the Remainder Theorem to factorise the following expression : $2x^3 + x^2 - 13x + 6$. (2010)

(ii) Using the Remainder Theorem, factorise completely the following polynomial: $3x^2 + 2x^2 - 19x + 6$ (2012)

Solution:

(i) Let $f(x) = 2x^3 + x^2 - 13x + 6$

Factors of 6 are ±1, ±2, ±3, ±6

Let x = 2, then

$$f(2) = 2(2)^3 + (2)^2 - 13 \times 2 + 6$$
$$= 16 + 4 - 26 + 6 = 26 - 26 = 0$$
$$\because f(2) = 0$$
$$\therefore x - 2 \text{ is the factor of } f(x)$$

(By Remainder Theorem)

Dividing $f(x)$ by $x - 2$, we get

$$\begin{array}{r|l} x-2 & 2x^3 + x^2 - 13x + 6 \ (2x^2 + 5x - 3 \\ & 2x^3 - 4x^2 \\ & \underline{-\quad +} \\ & \quad 5x^2 - 13x \\ & \quad 5x^2 - 10x \\ & \quad \underline{-\quad +} \\ & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ & \qquad \underline{+\quad -} \\ & \qquad \times \end{array}$$

$$\therefore f(x) = (x-2)(2x^2 + 5x - 3)$$
$$= (x-2)\{2x^2 + 6x - x - 3\}$$
$$= (x-2)\{2x(x+3) - 1(x+3)\}$$
$$= (x-2)(x+3)(2x-1)$$

(ii) $P(x) = 3x^3 + 2x^2 - 19x + 6$

$P(1) = 3 + 2 - 19 + 6 = -8 \neq 0$

$P(-1) = -3 + 2 + 19 + 6 = -24 \neq 0$

$P(2) = 24 + 8 - 38 + 6 = 0$

Hence, $(x - 2)$ is a factor of $P(x)$

$$\therefore P(x) = 3x^3 + 2x^2 - 19x + 6$$
$$= 3x^3 - 6x^2 + 8x^2 - 16x - 3x + 6$$
$$= 3x^2(x-2) + 8x(x-2) - 3(x-2)$$
$$= (x-2)(3x^2 + 8x - 3) = (x-2)(3x^2 + 9x - x - 3)$$
$$= (x-2)\{3x(x+3) - 1(x+3) = (x-2)(x+3)(3x-1)$$

Question 17.
Using the Remainder and Factor Theorem, factorize the following polynomial:
$x^3 + 10x^2 - 37x + 26$.

Solution:

$f(x) = x^3 + 10x^2 - 37x + 26$

$f(1) = (1)^3 + 10(1)^2 - 37(1) + 26$

$= 1 + 10 - 37 + 26 = 0$

$x = 1$

$$
\begin{array}{r|l}
x-1 & x^3 + 10x^2 - 37x + 26 \quad (x^2 + 11x - 26 \\
 & \underline{\begin{array}{l} x^3 - x^2 \\ - \quad + \end{array}} \\
 & \quad 11x^2 - 37x \\
 & \quad \underline{\begin{array}{l} 11x^2 - 11x \\ - \quad\quad + \end{array}} \\
 & \quad\quad -26x + 26 \\
 & \quad\quad \underline{\begin{array}{l} -26x + 26 \\ + \quad\quad - \end{array}} \\
 & \quad\quad\quad \underline{0}
\end{array}
$$

$x - 1$ is factor of $f(x)$

$\therefore f(x) = (x - 1)(x^2 + 11x - 26)$

$= (x - 1)(x^2 + 13x - 2x - 26)$

$= (x - 1)[x(x + 13) - 2(x + 13)]$

$= (x - 1)[(x - 2)(x + 13)]$

Question 18.

If $(2x + 1)$ is a factor of $6x^3 + 5x^2 + ax - 2$ find the value of a

Solution:

Let 2x + 1 = 0, then x = $-\frac{1}{2}$

Substituting the value of x in f(x),

f(x) = $6x^3 + 5x^2 + ax - 2$

$$f\left(-\frac{1}{2}\right)=6\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)^2+a\left(-\frac{1}{2}\right)-2$$

$$=6\left(-\frac{1}{8}\right)+5\left(\frac{1}{4}\right)+a\left(-\frac{1}{2}\right)-2$$

$$=-\frac{3}{4}+\frac{5}{4}-\frac{a}{2}-2=\frac{-3+5-2a-8}{4}=\frac{-6-2a}{4}$$

$\because$ $2x+1$ is a factor of $f(x)$

$\therefore$ Remainder = 0

$$\therefore \frac{-6-2a}{4}=0 \Rightarrow -6-2a=0$$

$$\Rightarrow 2a=-6 \Rightarrow a=-3$$

$\therefore$ $a=-3$

Question 19.

If (3x – 2) is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k.

Solution:

Let 3x − 2 = 0, then 3x = 2

$\Rightarrow$ x = $\frac{2}{3}$

Substituting the value of x in f(x),

f(x) = $3x^3 - kx^2 + 21x - 10$

$$f\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^3-k\left(\frac{2}{3}\right)^2+21\left(\frac{2}{3}\right)-10$$

$$=3\times\frac{8}{27}-k\times\frac{4}{9}+21\times\frac{2}{3}-10$$

$$=\frac{8}{9}-\frac{4k}{9}+14-10=\frac{8-4k}{9}+4$$

$\because$ Remainder is 0

$$\therefore \frac{8-4k}{9}+4=0$$

$$\Rightarrow 8-4k+36=0$$

$$\Rightarrow -4k+44=0 \Rightarrow 4k=44$$

$\therefore$ $k=11$

Question 20.
If $(x - 2)$ is a factor of $2x^3 - x^2 + px - 2$, then
(i) find the value of p.
(ii) with this value of p, factorize the above expression completely
Solution:
(i) Let $x - 2 = 0$, then $x = 2$
Now $f(x) = 2x^3 - x^2 + px - 2$

$\therefore \quad f(2) = 2(2)^3 - (2)^2 + p \times 2 - 2$
$= 2 \times 8 - 4 + 2p - 2 \quad = 16 - 4 + 2p - 2$
$= 10 + 2p$

(ii) $\therefore f(2) = 0$, then $10 + 2p = 0$
$\Rightarrow \quad 2p = -10 \quad \Rightarrow \quad p = -5$
Now, the polynomial will be
$2x^3 - x^2 - 5x - 2$
$= (x - 2)(2x^2 + 3x + 1) = (x - 2)[2x^2 + 2x + x + 1]$
$= (x - 2)[2x(x + 1) + 1(x + 1)]$
$= \mathbf{(x - 2)(x + 1)(2x + 1)}$

$$
\begin{array}{r|l}
x - 2 & 2x^3 - x^2 - 5x - 2 \;(\; 2x^2 + 3x + 1 \\
 & 2x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & 3x^2 - 5x \\
 & 3x^2 - 6x \\
 & - \quad + \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & \times \\
\hline
\end{array}
$$

Question 21.
Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation, $(K + 2)x^2 - Kx + 6 = 0$.
Also, find the other root of the equation.

Solution:

$(K + 2)x^2 - Kx + 6 = 0$ ...(1)

Substitute x = 3 in equation (1)

$$(-4 + 2)x^2 - (-4)x + 6 = 0$$
$$\Rightarrow -2x^2 + 4x + 6 = 0$$
$$\Rightarrow x^2 - 2x - 3 = 0 \qquad \text{(Dividing by 2)}$$
$$\Rightarrow x^2 - 3x + x - 3 = 0 \Rightarrow x(x - 3) + 1(x - 3) = 0$$
$$\Rightarrow (x + 1)(x - 3) = 0$$

So, the roots are $x = -1$ and $x = 3$

Thus, the other root of the equation is $x = -1$

Question 22.

What number should be subtracted from $2x^3 - 5x^2 + 5x$ so that the resulting polynomial has $2x - 3$ as a factor?

Solution:

Let the number to be subtracted be k and the resulting polynomial be f(x), then

$f(x) = 2x^3 - 5x^2 + 5x - k$

Since, $2x - 3$ is a factor of f(x),

Now, converting $2x - 3$ to factor theorem

$$f\left(\frac{3}{2}\right) = 0$$
$$\Rightarrow 2x^3 - 5x^2 + 5x - k = 0$$
$$\Rightarrow 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) - k = 0$$
$$\Rightarrow 2 \times \frac{27}{8} - 5 \times \frac{9}{4} + 5 \times \frac{3}{2} - k = 0 \Rightarrow \frac{27}{4} - \frac{45}{4} + \frac{15}{2} - k = 0$$
$$\Rightarrow 27 - 45 + 30 - 4k = 0 \Rightarrow -4k + 12 = 0$$
$$\Rightarrow k = \frac{-12}{-4}$$
$$\Rightarrow k = 3$$

Question 23.

Find the value of the constants a and b, if $(x - 2)$ and $(x + 3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$.

Solution:

Let x − 2 = 0, then x = 0

Substituting value of x in f(x)

$f(x) = x^3 + ax^2 + bx - 12$

$f(2) = (2)^3 + a(2)^2 + b(2) - 12$

$= 8 + 4a + 2b - 12 = 4a + 2b - 4$

$\because$ $x - 2$ is a factor

$\therefore$ $4a + 2b - 4 = 0 \Rightarrow 4a + 2b = 4$

$\Rightarrow$ $2a + b = 2$

Again let $x + 3 = 0$, then $x = -3$

Substituting the value of x in $f(x)$

$f(x) = x^3 + ax^2 + bx - 12$

$= (-3)^3 + a(-3)^2 + b(-3) - 12$

$= -27 + 9a - 3b - 12 = -39 + 9a - 3b$

$\because$ $x + 3$ is a factor of $f(x)$

$\therefore$ $-39 + 9a - 3b = 0 \Rightarrow 9a - 3b = 39$

$\Rightarrow$ $3a - b = 13$ ...(*ii*)

Adding (*i*) and (*ii*)

$5a = 15 \Rightarrow a = 3$

Substituting the value of a in (*i*)

$2(3) + b = 2 \Rightarrow 6 + b = 2$

$\Rightarrow$ $b = 2 - 6$

$\therefore$ $b = -4$

Hence $a = 3$, $b = -4$

Question 24.

If (x + 2) and (x – 3) are factors of x^3 + ax + b, find the values of a and b. With these values of a and b, factorize the given expression.

Solution:

Let x + 2 = 0, then x = -2

Substituting the value of x in f(x),

$f(x) = x^3 + ax + b$

$f(-2) = (-2)^3 + a(-2) + b = -8 - 2a + b$

$\because x + 2$ is a factor

$\therefore$ Remainder is zero.

$\therefore -8 - 2a + b = 0$

$\Rightarrow -2a + b = 8$

$\therefore 2a - b = -8 \qquad ...(i)$

Again let $x - 3 = 0$, then $x = 3$,

Substituting the value of x in $f(x)$,

$f(x) = x^3 + ax + b$

$f(3) = (3)^3 + a(3) + b = 27 + 3a + b$

$\because x - 3$ is a factor $\quad \therefore$ Remainder $= 0$

$\Rightarrow 27 + 3a + b = 0$

$\Rightarrow 3a + b = -27 \qquad ...(ii)$

Adding (*i*) and (*ii*)

$5a = -35 \Rightarrow a = \frac{-35}{5} = -7$

Substituting value of a in (i)

$2(-7) - b = -8 \Rightarrow -14 - b = -8$

$-b = -8 + 14 \Rightarrow -b = 6 \quad \therefore b = -6$

Hence $a = -7, b = -6$

$(x + 2)$ and $(x - 3)$ are the factors of

$x^3 + ax + b \Rightarrow x^3 - 7x - 6$

Now dividing $x^3 - 7x - 6$ by $(x + 2)$ $(x - 3)$ or $x^2 - x - 6$, we get

$$
\begin{array}{r|l}
x^2 - x - 6 & x^3 - 7x - 6 \; (x + 1 \\
 & x^3 - x^2 - 6x \\
 & \underline{- \quad + \quad +} \\
 & \quad x^2 - x - 6 \\
 & \quad x^2 - x - 6 \\
 & \quad \underline{- \quad + \quad +} \\
 & \quad \times
\end{array}
$$

$\therefore$ Factors are $(x + 2)$, $(x - 3)$ and $(x + 1)$

Question 25.

$(x - 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$. When this expression is divided by $(x - 3)$, it leaves the remainder 3. Find the values of a and b. (2005)

Solution:

As $x - 2$ is a factor of

$f(x) = x^3 + ax^2 + bx + 6$

$\therefore f(2) = 0$

$\therefore (2)^3 + a(2)^2 + b(2) + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\Rightarrow 4a + 2b = -14$

$\Rightarrow 2a + b = -7 \quad \text{....}(i)$

as on dividing $f(x)$ by $x - 3$

remainder = 3

$\therefore f(3) = 3$

$\therefore (3)^3 + a(3)^2 + b(3) + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\Rightarrow 9a + 3b = -30$

$\Rightarrow 3a + b = -10 \quad \text{....}(ii)$

Solving simultaneously equation (*i*) and (*ii*),

$$\begin{aligned} \therefore \quad 2a + b &= -7 \\ 3a + b &= -10 \\ \text{Subtracting,} \quad - \quad - \quad & \quad + \\ -a &= 3 \\ a &= -3 \end{aligned}$$

Substituting value of a in equation (*i*)

$2(-3) + b = -7$

$\therefore -6 + b = -7$

$\therefore b = -1$

$\therefore a = -3, b = -1$

Question 26.

If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b.

Solution:

f(x) = $2x^3 + ax^2 + bx - 14$

$\therefore$ $(x - 2)$ is factor of f(x)

$f(2) = 0$

$2(2)^3 + a(2)^2 + b(2) - 14 = 0$

$16 + 4a + 2b - 14 = 0 \Rightarrow 4a + 2b = -2$

$2a + b = -1 \quad \ldots(i)$

Also, $(x - 3)$ it leaves remainder = 52

$\therefore f(3) = 52$

$2(3)^3 + a(3)^2 + b(3) - 14 = 52$

$\Rightarrow 54 + 9a + 3b - 14 = 52 \Rightarrow 9a + 3b = 52 - 40$

$9a + 3b = 12$

$3a + b = 4 \quad \ldots(ii)$

From (*i*) and (*ii*)

$$\begin{array}{rcl} 2a + b & = & -1 \\ 3a + b & = & 4 \\ - \quad - & & - \\ \hline -a & = & -5 \\ \hline \end{array}$$

Subtracting

$\therefore a = 5$ put in (*i*)

$\therefore 2(5) + b = -1 \Rightarrow b = -1 - 10 \Rightarrow b = -11$

$\therefore a = 5, b = -11$

$$\Rightarrow \frac{-27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3 = 0$$

$\Rightarrow -27a + 54 - 12b - 24 = 0$ (Multiplying by 8)

$\Rightarrow -27a - 12b + 30 = 0 \Rightarrow -27a - 12b = -30$

$\Rightarrow 9a + 4b = 10$ [Dividing by (–3)]

$9a + 4b = 10$ ...(*i*)

Again let $x + 2 = 0$ then $x = -2$

Substituting the value of x in $f(x)$

$f(x) = ax^3 + 3x^2 + bx - 3$

$f(-2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$

$= -8a + 12 - 2b - 3 = -8a - 2b + 9$

$\because$ Remainder $= -3$

$\therefore -8a - 2b + 9 = -3 \Rightarrow -8a - 2b = -3 - 9$

$\Rightarrow -8a - 2b = -12$ (Dividing by 2)

$\Rightarrow 4a + b = 6$ ...(*ii*)

Multiplying (*ii*) by 4

$$\begin{array}{rcl} 16a + 4b &=& 24 \\ 9a + 4b &=& 10 \\ - \quad - & & - \\ \hline 7a &=& 14 \\ \hline \end{array}$$

Substracting,

$7a = 14 \Rightarrow a = \frac{14}{7} = 2.$

Substituting the value of a in (i)

$9(2) + 4b = 10 \Rightarrow 18 + 4b = 10$

$\Rightarrow 4b = 10 - 18 \Rightarrow 4b = -8$

$\therefore b = \frac{-8}{4} = -2$

Hence $a = 2, b = -2$

$\therefore f(x) = ax^3 + 3x^2 + bx - 3 = 2x^3 + 3x^2 - 2x - 3$

$\because 2x + 3$ is a factor

$\therefore$ Dividing $f(x)$ by $x + 2$

$$
\begin{array}{r|l}
2x+3 & 2x^3 + 3x^2 - 2x - 3 \;(x^2 - 1 \\
 & 2x^3 + 3x^2 \\
 & - \quad\; - \\
\hline
 & -2x - 3 \\
 & -2x - 3 \\
 & + \quad\; + \\
\hline
 & \times
\end{array}
$$

$\therefore 2x^3 + 3x^2 - 2x - 3$

$= (2x + 3)(x^2 - 1) = (2x + 3)[(x^2) - (1)^2]$

$= (2x + 3)(x + 1)(x - 1)$

Question 27.

If $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$ and leaves remainder -3 when divided by $(x + 2)$, find the values of a and 6. With these values of a and 6, factorize the given expression.

Solution:

Let 2x + 3 = 0 then 2x = -3

$\Rightarrow x = \frac{-3}{2}$

Substituting the value of x in f(x),

$f(x) = ax^3 + 3x^2 + 6x - 3$

$$f\left(\frac{-3}{2}\right) = a\left(\frac{-3}{2}\right)^3 + 3\left(\frac{-3}{2}\right)^2 + b\left(\frac{-3}{2}\right) - 3$$

$$= a\left(\frac{-27}{8}\right) + 3\left(\frac{9}{4}\right) + b\left(\frac{-3}{2}\right) - 3$$

$$= \frac{-27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$$

$\because$ 2 x + 3 is a factor of $f(x)$

$\therefore$ Remainder = 0

Question 28.

Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If x – 2 is a factor of f(x) but leaves the remainder – 15 when it divides g(x), find the values of a and b. With these values of a and b, factorise the expression. $f(x) + g(x) + 4x^2 + 7x$.

Solution:

$f(x) = ax^2 + bx + 2$

$g(x) = bx^2 + ax + 1$

$x - 2$ is a factor of $f(x)$

Let $x - 2 = 0$

$\Rightarrow x = 2$

$\therefore f(2) = a(2)^2 + b \times 2 + 2 = 4a + 2b + 2$

$\therefore 4a + 2b + 2 = 0$ $\quad$ ($\because x - 2$ is its factor)

$\Rightarrow 2a + b + 1 = 0$ $\quad$ ....(*i*) $\quad$ (Dividing by 2)

Dividing $g(x)$ by $x - 2$, remainder $= -15$

Let $x - 2 = 0 \Rightarrow x = 2$

$\therefore g(2) = b(2)^2 + a \times 2 + 1$

$= 4b + 2a + 1$

$\because$ Remainder is -15

$\therefore 4b + 2a + 1 = -15 \Rightarrow 4b + 2a + 1 + 15 = 0$

$\Rightarrow 4b + 2a + 16 = 0 \Rightarrow 2b + a + 8 = 0$

(Dividing by 2)

$\Rightarrow a + 2b + 8 = 0$ $\quad$ ....(*ii*)

Multiplying (*i*) by 2 and (*ii*) by 1

$$\begin{array}{r} 4a + 2b + 2 = 0 \\ a + 2b + 8 = 0 \\ - \quad - \quad\quad - \\ \hline 3a \quad\quad\quad - 6 = 0 \end{array} \Rightarrow 3a = 6 \Rightarrow a = \frac{6}{3}$$

$\therefore a = 2$

Substituting the value of a in (*i*)

$2 \times 2 + b + 1 = 0 \Rightarrow 4 + b + 1 = 0$

$\Rightarrow b + 5 = 0 \Rightarrow b = -5$

Hence $a = 2$, $b = -5$

Now $f(x) + g(x) = 4x^2 + 7x$

$= 2x^2 - 5x + 2 + (-5x^2 + 2x + 1) + 4x^2 + 7x$

$= 2x^2 - 5x + 2 - 5x^2 + 2x + 1 + 4x^2 + 7x$

$= 6x^2 - 5x^2 - 5x + 2x + 7x + 2 + 1$

$= x^2 + 2x + 3$

$= x^2 + x + 3x + 3$

$= x(x + 1) + 3(x + 1) = (x + 1)(x + 3)$