Chapter 5 Quadratic Equations in One Variable Ex 5.3

Solve the following (1 to 8) equations by using the formula:

Question 1.
(i) $2x^2 - 7x + 6 = 0$
(ii) $2x^2 - 6x + 3 = 0$
Solution:

(i) $2x^2 - 7x + 6 = 0$

Here a = 2, b = -7, c = 6

$$\therefore D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 6$$
$$= 49 - 48 = 1$$
$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{1}}{2 \times 2} = \frac{7 \pm 1}{4}$$
$$\therefore x_1 = \frac{7+1}{4} = \frac{8}{4} = 2 \text{ and } x_2 = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2}$$
$$\therefore x = 2, 3/2$$

(ii) $2x^2 - 6x + 3 = 0$

Here $a = 2, b = -6, c = 3$

then $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3$
$= 36 - 24 = 12$

$$\text{Now } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4}$$
$$\therefore x_1 = \frac{6 + 2\sqrt{3}}{4} = \frac{2(3+\sqrt{3})}{4} = \frac{3+\sqrt{3}}{2}$$
$$x_2 = \frac{6 - 2\sqrt{3}}{4} = \frac{2(3-\sqrt{3})}{4} = \frac{3-\sqrt{3}}{2}$$
$$\text{Hence } x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$$

Question 2.
(i) $x^2 + 7x - 7 = 0$
(ii) $(2x + 3)(3x - 2) + 2 = 0$

Solution:

(i) $x^2 + 7x - 7 = 0$

Here a = 1, b = 7, c = -7

$$\therefore D = b^2 - 4ac = (7)^2 - 4 \times 1 (-7)$$
$$= 49 + 28 = 77$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{77}}{2 \times 1} = \frac{-7 \pm \sqrt{77}}{2}$$

$$\therefore x_1 = \frac{-7 + \sqrt{77}}{2} \text{ and } x_2 = \frac{-7 - \sqrt{77}}{2}$$

$$\text{Hence } x = \frac{-7 + \sqrt{77}}{2}, \frac{-7 - \sqrt{77}}{2}$$

(ii) $(2x + 3)(3x - 2) + 2 = 0$

$$6x^2 - 4x + 9x - 6 + 2 = 0$$
$$6x^2 + 5x - 4 = 0$$

Here $a = 6, b = 5, c = -4$

$$D = b^2 - 4ac = (5)^2 - 4 \times 6 \times (-4)$$
$$= 25 + 96 = 121$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{121}}{2 \times 6} = \frac{-5 \pm 11}{12}$$

$$\therefore x_1 = \frac{-5 + 11}{12} = \frac{6}{12} = \frac{1}{2}$$

$$x_2 = \frac{-5 - 11}{12} = \frac{-16}{12} = \frac{-4}{3}$$

$$\text{Hence } x = \frac{1}{2}, \frac{-4}{3}$$

Question 3.

(i) $256x^2 - 32x + 1 = 0$

(ii) $25x^2 + 30x + 7 = 0$

Solution:

(i) $256x^2 - 32x + 1 = 0$

Here a = 256, b = -32, c = 1

$$D = b^2 - 4ac = (-32)^2 - 4 \times 256 \times 1$$
$$= 1024 - 1024 = 0$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-32) \pm \sqrt{0}}{2 \times 256} = \frac{32}{512} = \frac{1}{16}$$

$$x_1 = \frac{1}{16}, x_2 = \frac{1}{16} \quad \text{Hence } x = \frac{1}{16}, \frac{1}{16}$$

(ii) $25x^2 + 30x + 7 = 0$

Here $a = 25, b = 30, c = 7$

$$D = b^2 - 4ac = (30)^2 - 4 \times 25 \times 7$$

$$= 900 - 700 = 200$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-30 \pm \sqrt{200}}{2 \times 25}$$

$$= \frac{-30 \pm \sqrt{100 \times 2}}{50} = \frac{-30 \pm 10\sqrt{2}}{50} = \frac{-3 \pm \sqrt{2}}{5}$$

$$\therefore x_1 = \frac{-3 + \sqrt{2}}{5} \text{ and } x_2 = \frac{-3 - \sqrt{2}}{5}$$

$$\text{Hence } x = \frac{-3 + \sqrt{2}}{5}, \frac{-3 - \sqrt{2}}{5}$$

Question 4.

(i) $2x^2 + \sqrt{5}x - 5 = 0$

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Solution:

(i) $2x^2 + \sqrt{5}x - 5 = 0$

Here a = 2, b = $\sqrt{5}$, c = -5

$$D = b^2 - 4ac = \left(\sqrt{5}\right)^2 - 4 \times 2 \times (-5)$$

$$= 5 + 40 = 45$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{5} \pm \sqrt{45}}{2 \times 2}$$

$$= \frac{-\sqrt{5} \pm \sqrt{9 \times 5}}{4} = \frac{-\sqrt{5} \pm 3\sqrt{5}}{4}$$

$$\therefore x_1 = \frac{-\sqrt{5} + 3\sqrt{5}}{4} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

$$x_2 = \frac{-\sqrt{5} - 3\sqrt{5}}{4} = \frac{-4\sqrt{5}}{4} = -\sqrt{5}$$

Hence $x = \frac{\sqrt{5}}{2}, -\sqrt{5}$

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Here $a = \sqrt{3}, b = 10, c = -8\sqrt{3}$

$$D = b^2 - 4ac = (10)^2 - 4 \times \sqrt{3} \times (-8\sqrt{3})$$

$$= 100 + 96 = 196$$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-10 \pm \sqrt{196}}{2 \times \sqrt{3}} = \frac{-10 \pm 14}{2\sqrt{3}}$$

$$\therefore x_1 = \frac{-10 + 14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$x_2 = \frac{-10 - 14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}} = \frac{-12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{-12\sqrt{3}}{3} = -4\sqrt{3}$$

Hence $x = \frac{2\sqrt{3}}{3}, -4\sqrt{3}$

Question 5.

(i) $\frac{x-2}{x+2}+\frac{x+2}{x-2}=4$

(ii) $\frac{x+1}{x+3}=\frac{3x+2}{2x+3}$

Solution:

(i) $\frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$

$\Rightarrow \frac{(x-2)^2+(x+2)^2}{(x+2)(x-2)} = 4$

$\Rightarrow \frac{x^2-4x+4+x^2+4x+4}{x^2-4} = 4$

$\Rightarrow 2x^2 + 8 = 4x^2 - 16$

$\Rightarrow 2x^2 + 8 - 4x^2 + 16 = 0$

$\Rightarrow -2x^2 + 24 = 0 \quad \Rightarrow \quad x^2 - 12 = 0$

Here $a = 1, b = 0, c = -12$

$D = b^2 - 4ac = (0)^2 - 4 \times 1 (-12)$

$= 0 + 48 = 48$

$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{0 \pm \sqrt{48}}{2\times 1} = \frac{\pm\sqrt{48}}{2}$

$= \frac{\pm\sqrt{16\times 3}}{2} = \pm \frac{4\sqrt{3}}{2} = \pm 2\sqrt{3}$

Hence roots are $2\sqrt{3}, -2\sqrt{3}$

(ii) $\frac{x+1}{x+3} = \frac{3x+2}{2x+3}$

$(x + 1)(2x + 3) = (3x + 2)(x + 3)$

$\Rightarrow 2x^2 + 3x + 2x + 3 = 3x^2 + 9x + 2x + 6$

$\Rightarrow 2x^2 + 5x + 3 - 3x^2 - 11x - 6 = 0$

$\Rightarrow -x^2 - 6x - 3 = 0 \Rightarrow x^2 + 6x + 3 = 0$

Here $a = 1, b = 6, c = 3$

$D = b^2 - 4ac = (6)^2 - 4 \times 1 \times 3$

$= 36 - 12 = 24$

$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-6 \pm \sqrt{24}}{2 \times 1}$

$= \frac{6 \pm \sqrt{4 \times 6}}{2} = \frac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

$\therefore x_1 = -3 + \sqrt{6}, x_2 = -3 - \sqrt{6}$

Hence $x = -3 + \sqrt{6}, -3 - \sqrt{6}$

Question 6.

(i) $a(x^2 + 1) = (a^2 + 1)x, a \neq 0$

(ii) $4x^2 - 4ax + (a^2 - b^2) = 0$

Solution:

(i) $a(x^2 + 1) = (a^2 + 1)x$

$ax^2 - (a^2 + 1)x + a = 0$

Here $a = a, b = -(a^2 + 1), c = a$

$D = b^2 - 4ac = [-(a^2 + 1)]^2 - 4 \times a \times a$

$= a^4 + 2a^2 + 1 - 4a^2 = a^4 - 2a^2 + 1$

$= (a^2 - 1)^2$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{(a^2+1) \pm \sqrt{(a^2-1)^2}}{2 \times a} = \frac{(a^2+1) \pm (a^2-1)}{2a}$

$\therefore x_1 = \frac{a^2+1+a^2-1}{2a} = \frac{2a^2}{2a} = a$

$x_2 = \frac{a^2+1-a^2+1}{2a} = \frac{2}{2a} = \frac{1}{a}$

Hence $x = a, \frac{1}{a}$

(ii) $4x^2 - 4ax + (a^2 - b^2) = 0$

Here $a = 4, b = -4a, c = a^2 - b^2$

$D = b^2 - 4ac = (-4a)^2 - 4 \times 4(a^2 - b^2)$

$= 16a^2 - 16(a^2 - b^2)$

$= 16a^2 - 16a^2 + 16b^2$

$D = 16b^2$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4a) \pm \sqrt{16b^2}}{2 \times 4}$

$= \frac{4a \pm 4b}{8} = \frac{a \pm b}{2}$

$\therefore x_1 = \frac{a+b}{2}, x_2 = \frac{a-b}{2}$

Hence $x = \frac{a+b}{2}, \frac{a-b}{2}$

Question 7.

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x} + \frac{1}{x-2} = 3, x \neq 0, 2$

Solution:

(i) $x - \frac{1}{x} = 3$

$x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

Here $a = 1, b = -3, c = -1$

$\therefore b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1)$

$= 9 + 4 = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{13}}{2 \times 1} = \frac{3 \pm \sqrt{13}}{2}$$

$$\therefore x = \frac{3 + \sqrt{13}}{2} \text{ and } \frac{3 - \sqrt{13}}{2}$$

(ii) $\frac{1}{x} + \frac{1}{x-2} = 3$

$$\frac{x - 2 + x}{x(x-2)} = 3 \Rightarrow \frac{2x - 2}{x^2 - 2x} = 3$$

$\Rightarrow 3x^2 - 6x = 2x - 2 \Rightarrow 3x^2 - 6x - 2x + 2 = 0$

$\Rightarrow 3x^2 - 8x + 2 = 0$

Here $a = 3, b = -8, c = 2$

$b^2 - 4ac = (-8)^2 - 4 \times 3 \times 2$

$= 64 - 24 = 40$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{40}}{2 \times 3} = \frac{8 \pm 2\sqrt{10}}{6} = \frac{4 \pm \sqrt{10}}{3}$$

$$\therefore x = \frac{4 + \sqrt{10}}{3} \text{ and } \frac{4 - \sqrt{10}}{3}$$

Question 8.

$\frac{1}{x-2} + \frac{1}{x-3} + \frac{1}{x-4} = 0$

Solution:

$\frac{1}{x-2}+\frac{1}{x-3}+\frac{1}{x-4}=0$

$\Rightarrow \frac{1}{x-2}+\frac{1}{x-3}=-\frac{1}{x-4}$

$$\Rightarrow \frac{x-3+x-2}{(x-2)(x-3)}=-\frac{1}{x-4}$$

$$\Rightarrow \frac{2x-5}{x^2-5x+6}=\frac{-1}{x-4}$$

$$(2x-5)(x-4) = -1(x^2-5x+6)$$
$$\Rightarrow 2x^2-8x-5x+20 = -x^2+5x-6$$
$$\Rightarrow 2x^2-8x-5x+20+x^2-5x+6=0$$
$$\Rightarrow 3x^2-18x+26=0$$

Here, $a=3, b=-18, c=26$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\frac{1}{x-2}+\frac{1}{x-3}+\frac{1}{x-4}=0$

$\Rightarrow \frac{1}{x-2}+\frac{1}{x-3}=-\frac{1}{x-4}$

$$\Rightarrow \frac{x-3+x-2}{(x-2)(x-3)}=-\frac{1}{x-4}$$

$$\Rightarrow \frac{2x-5}{x^2-5x+6}=\frac{-1}{x-4}$$

$$(2x-5)(x-4) = -1(x^2-5x+6)$$
$$\Rightarrow 2x^2-8x-5x+20 = -x^2+5x-6$$
$$\Rightarrow 2x^2-8x-5x+20+x^2-5x+6=0$$
$$\Rightarrow 3x^2-18x+26=0$$

Here, $a=3, b=-18, c=26$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Question 9.

Solve for $x: 2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5, x\neq -3, \frac{1}{2}$

Solution:

$x : 2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$

Let $\frac{2x-1}{x+3} = y$ then $\frac{x+3}{2x-1} = \frac{1}{y}$

$$\therefore\ 2y - \frac{3}{y} = 5$$

$2y^2 - 3 = 5y \Rightarrow 2y^2 - 5y - 3 = 0$

Here, $a = 2$, $b = -5$, $c = -3$

$b^2 - 4ac = (-5)^2 - 4 \times 2 \times (-3)$

$= 25 + 24 = 49$

$$\text{Now, } y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{49}}{2 \times 2} = \frac{5 \pm 7}{4}$$

$$y = \frac{5+7}{4} = \frac{12}{4} = 3$$

$$\text{or } y = \frac{5-7}{4} = \frac{-2}{4} = \frac{-1}{2}$$

$\therefore\ y = 3, \frac{-1}{2}$

When $y = 3$, then $\frac{2x-1}{x+3} = 3$

$\Rightarrow\ 3x + 9 = 2x - 1$

$\Rightarrow\ 3x - 2x = -1 - 9 \Rightarrow x = -10$

When $y = \frac{-1}{2}$, then

or $\frac{2x-1}{x+3} = \frac{-1}{2}$

$4x - 2 = -x - 3$

$4x + x = -3 + 2 \Rightarrow 5x = -1$

$x = \frac{-1}{5}$

$\therefore\ x = -10, \frac{-1}{5}$

Question 10.

Solve the following equation by using quadratic equations for x and give your

(i) $x^2 - 5x - 10 = 0$

(ii) $5x(x + 2) = 3$

Solution:

(i) $x^2 - 5x - 10 = 0$

On comparing with, $ax^2 + bx + c = 0$

$$a = 1, b = -5, c = -10$$

$$\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2 \times 1}$$

$$\therefore x = \frac{5 \pm \sqrt{25 + 40}}{2}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8.06}{2}$$

$$\text{Either } x = \frac{5 + 8.06}{2} = \frac{13.06}{2} = 6.53$$

$$\text{or } x = \frac{5 - 8.06}{2} = \frac{-3.06}{2} = -1.53$$

$$\therefore x = 6.53, x = -1.53$$

(*ii*) $5x(x+2) = 3$

$5x(x+2) = 3$

$\Rightarrow 5x^2 + 10x = 3$

$\Rightarrow 5x^2 + 10x - 3 = 0$

Here $a = 5, b = 10, c = -3$

$D = b^2 - 4ac = (10)^2 - 4 \times 5 \times (-3)$

$= 100 + 60 = 160$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{160}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{16 \times 10}}{10} = \frac{-10 \pm 4\sqrt{10}}{10}$$

$$= \frac{-10 \pm 4(3.162)}{10} = \frac{-10 \pm 12.648}{10}$$

$$\therefore x_1 = \frac{-10 + 12.648}{10} = \frac{2.648}{10} = 0.2648$$

$$= 0.265$$

$$x_2 = \frac{-10 - 12.648}{10} = \frac{-22.648}{10} = -2.2648$$

$\therefore x = 0.26, -2.26$

Question 11.

Solve the following equations by using quadratic formula and give your answer correct to 2 decimal places :

(i) $4x^2 - 5x - 3 = 0$

(ii) $2x - \frac{1}{x} = 1$

Solution:

(i) Given equation $4x^2 - 5x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we have

$a = 4, b = -5, c = -3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{8} = \frac{5 \pm \sqrt{73}}{8} = \frac{5 \pm 8.544}{8}$$

$$= \frac{5 + 8.544}{8} \text{ or } \frac{5 - 8.544}{8}$$

$$= \frac{13.544}{8} \text{ or } \frac{-3.544}{8}$$

$= 1.693$ or -0.443

$= 1.69$ or -0.44 (correct to 2 decimal places)

(ii) $2x - \frac{1}{x} = 7 \Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$ ....(i)

Comparing (i) with $ax^2 + bx + c$, we get,

$a = 2, b = -7, c = -1$

$\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2) \times (-1)}}{2 \times 2}$

$\Rightarrow \frac{7 \pm \sqrt{49 + 8}}{4} = \frac{7 \pm \sqrt{57}}{4}$

$\Rightarrow x = \frac{7 + \sqrt{57}}{4}$ or $x = \frac{7 - \sqrt{57}}{4}$

$\Rightarrow x = \frac{7 + 7.55}{4}$ or $x = \frac{7 - 7.55}{4}$

$\Rightarrow x = \frac{14.55}{4}$ or $x = \frac{-0.55}{4}$

$\Rightarrow x = 3.64$ or $x = -0.14$

Question 12.

Solve the following equation: $x - 18x = 6$. Give your answer correct to two x significant figures. (2011)

Solution:

$x - \frac{18}{x} = 6$

$\Rightarrow x^2 - 6x - 18 = 0$

a = 1, b = -6, c = -18

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 + 72}}{2}$$

$$= \frac{6 \pm \sqrt{108}}{2} = \frac{6 \pm 6\sqrt{3}}{2} = \text{ or } \frac{6(1 - 1.73)}{2}$$

$$= 3 \times 2.73 \text{ or } 3 \times -0.73 = 8.19 \text{ or } -2.19$$

Question 13.

Solve the equation $5x^2 - 3x - 4 = 0$ and give your answer correct to 3 significant figures:

Solution:

We have $5x^2 - 3x - 4 = 0$

Here a = 5, b = − 3, c = − 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 4 \times 5 \times 4}}{2 \times 5} = \frac{3 \pm \sqrt{89}}{10}$$

$$x = \frac{3 + 9.43}{10} \text{ or } x = \frac{3 - 9.43}{10}$$

$$\Rightarrow x = \frac{12.43}{10} \quad \text{or } x = \frac{-6.43}{10}$$

$$\Rightarrow x = 1.24 \quad \text{or } x = -0.643$$