Chapter 5 Quadratic Equations in One Variable Ex 5.1

ML Aggarwal Class 10 Solutions for ICSE Maths Chapter 5 Quadratic Equations in One Variable Ex 5.1

Question 1.
Check whether the following are quadratic equations:
(i) $3-\sqrt{x^2-2x+35}=0$
(ii) (2x + 1) (3x – 2) = 6(x + 1) (x – 2)
(iii) $(x-3)^3+5=x^3+7x^2-1$
(iv) $x-\frac{3}{x}=2, x\neq 0$
(v) $x+\frac{2}{x}=x^2, x\neq 0$
(vi) $x^2+\frac{1}{x^2}=3, x\neq 0$
Solution:

(i) $\sqrt{3}x^2 - 2x + \frac{3}{5} = 0$

It is a quadratic equation as it is power of 2.

(ii) $(2x + 1)(3x - 2) = 6(x + 1)(x - 2)$

$6x^2 - 4x + 3x - 2 = 6(x^2 - 2x + x - 2)$

$6x^2 - x - 2 = 6x^2 - 12x + 6x - 12$

$12x - 6x - x = -12 + 2$

$5x = -10$

It is not a quadratic equation.

(iii) $(x - 3)^3 + 5 = x^3 + 7x^2 - 1$

$x^3 - 3x^2 \times 3 + 3x \times 9 - 27 + 5 = x^3 + 7x^2 - 1$

$-9x^2 + 27x - 22 - 7x^2 + 1 = 0$

$-16x^2 + 27x - 21 = 0$

$\Rightarrow 16x^2 - 27x + 21 = 0$

It is a quadratic equation.

(iv) $x - \frac{3}{x} = 2, x \neq 0$

$x^2 - 3 = 2x \Rightarrow x^2 - 2x - 3 = 0$

It is a quadratic equation.

(v) $x + \frac{2}{x} = x^2, x \neq 0$

$x^2 + 2 = x^3 \Rightarrow x^3 - x^2 - 2 = 0$

It is not a quadratic equation.

(vi) $x^2 + \frac{1}{x^2} = 3, x \neq 0$

$x^4 + 6 = 3x^2$

$x^4 - 3x^2 + 6 = 0$

It is not a quadratic equation.

Question 2.

In each of the following, determine whether the given numbers are roots of the given equations or not;

(i) $x^2 - x + 1 = 0$; 1, -1

(ii) $x^2 - 5x + 6 = 0$; 2, -3

(iii) $3x^2 - 13x - 10 = 0$; 5, $-\frac{2}{3}$

(iv) $6x^2 - x - 2 = 0$; $-\frac{1}{2}, \frac{2}{3}$

Solution:

(i) $x^2 - x + 1 = 0$; 1, -1

Where x = 1, then

$(1)^2 - 1 + 1 = 1 - 1 + 1 = 1 \neq 0$

$\therefore$ x = 1 does not satisfy it

and $(-1)^2 - (-1) + 1 = 0$

$1 + 1 + 1 \Rightarrow 3 \neq 0$

$\therefore$ $x = -1$, does not satisfy it

$\therefore$ $x = 1, -1$ are not roots of the equation.

(*ii*) $x^2 - 5x + 6 = 0$; 2, –3

When $x = 2$, then

$(2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 10 - 10 = 0$

$\therefore$ $x = 2$ is its root.

Where, $x = -3$, then

$(-3)^2 - 5(-3) + 6$

$= 9 + 15 + 6 = 30 \neq 0$

$\therefore$ $x = -3$ is not its solution

$\therefore$ 2 is root of the equation by –3 is not a root.

(iii) $3x^2 - 13x - 10 = 0$; 5, $\frac{-2}{3}$

$x = 5$,

$3(5)^2 - 13 \times 5 - 10 = 75 - 65 - 10$

$= 75 - 75 = 0$

$\therefore$ $x = 5$ is its root

If $x = \frac{-2}{3}$, then

$$3\left(\frac{-2}{3}\right)^2 - 13 \times \frac{-2}{3} - 10$$

$$= \frac{3 \times 4}{9} + \frac{26}{3} - 10$$

$$= \frac{4}{3} + \frac{26}{3} - 10 = \frac{30}{3} - 10 = 10 - 10 = 0$$

$\therefore$ $x = \frac{-2}{3}$ is also its root.

Hence both 5, $\frac{-2}{3}$ are its roots.

(*iv*) $6x^2 - x - 2 = 0; \frac{-1}{2}, \frac{2}{3}$

If $x = \frac{-1}{2}$, then

$= 6\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) - 2$

$= 6 \times \frac{1}{4} + \frac{1}{2} - 2 = \frac{3}{2} + \frac{1}{2} - 2$

$= \frac{4}{2} - 2 = 0$

$\therefore x = \frac{-1}{2}$ is its root

If $x = \frac{2}{3}$, then

$= 6 \times \frac{4}{9} - \frac{2}{3} - 2 = \frac{8}{3} - \frac{2}{3} - 2$

$= \frac{6}{3} - 2 = 0$

$\therefore x = \frac{2}{3}$ is also its root.

Hence $\frac{-1}{2}, \frac{2}{3}$ are both its root.

Question 3.
In each of the following, determine whether the given numbers are solutions of the given equation or not:
(i) $x^2 - 3\sqrt{3}x + 6 = 0; \sqrt{3}, -2\sqrt{3}$
(ii) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, 2\sqrt{2}$
Solution:

(i) $x^2 - 3\sqrt{3}x + 6 = 0$; $\sqrt{3}, -2\sqrt{3}$

(a) Substituting the value of $x = \sqrt{3}$

$$\text{L.H.S.} = x^2 - 3\sqrt{3}\,x + 6$$

$$= \left(\sqrt{3}\right)^2 - 3\sqrt{3} \times \sqrt{3} + 6 = 3 - 9 + 6 = 0$$

$$= \text{R.H.S.}$$

$\therefore\ x = \sqrt{3}$ is its solution

(b) $x = -2\sqrt{3}$

Substituting $x = -2\sqrt{3}$

$$\text{L.H.S.} = x^2 - 3\sqrt{3}\,x + 6$$

$$= \left(-2\sqrt{3}\right)^2 - 3\sqrt{3}\left(-2\sqrt{3}\right) + 6$$

$$= 12 + 18 + 6 = 36 \neq 0$$

$\because\ x = -2\sqrt{3}$ is not its solution

(*ii*) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}, 2\sqrt{2}$

(a) $x = -\sqrt{2}$

Substituting $x = -\sqrt{2}$

L.H.S. $= x^2 - \sqrt{2}x - 4$

$= \left(-\sqrt{2}\right)^2 - \sqrt{2}\left(-\sqrt{2}\right) - 4 = 2 + 2 - 4 = 0$

= R.H.S.

$\therefore$ $x = -\sqrt{2}$ is its solution

(b) $x = -2\sqrt{2}$

Substituting $x = -2\sqrt{2}$

L.H.S. $= x^2 - \sqrt{2}x - 4$

$= \left(-2\sqrt{2}\right)^2 - \sqrt{2}\left(-2\sqrt{2}\right) - 4$

$= 8 - 4 - 4 = 8 - 8 = 0$ = R.H.S.

$\therefore$ $x = -2\sqrt{2}$ is its solution

Question 4.

(i) If $-\frac{1}{2}$ is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k.

(ii) If $\frac{2}{3}$ is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k.

Solution:

(i) $x = -\frac{1}{2}$ is a solution of the

$3x^2 + 2kx - 3 = 0$,

Substituting the value of x in the given equation

$$3\left(\frac{-1}{2}\right)^2 + 2k\left(\frac{-1}{2}\right) - 3 = 0$$

$$3 \times \frac{1}{4} - k - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$\Rightarrow k = \frac{3}{4} - 3 = -\frac{9}{4}$$

Hence $k = -\frac{9}{4}$

(ii) $7x^2 + kx - 3 = 0, x = \frac{2}{3}$

$\because x = \frac{2}{3}$ is its solution

$\therefore 7\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 3 = 0$

$\Rightarrow 7 \times \frac{4}{9} + \frac{2}{3}k - 3 = 0$

$\Rightarrow \frac{28}{9} - 3 + \frac{2}{3}k = 0$

$\Rightarrow \frac{2}{3}k = 3 - \frac{28}{9}$

$\Rightarrow \frac{2}{3}k = \frac{27-28}{9} \Rightarrow \frac{2}{3}k = \frac{-1}{9}$

$\Rightarrow k = \frac{-1}{9} \times \frac{3}{2} = \frac{-1}{6}$

Hence $k = \frac{-1}{6}$

Question 5.

(i) If $\sqrt{2}$ is a root of the equation $kx^2 + \sqrt{2} - 4 = 0$, find the value of k.

(ii) If a is a root of the equation $x^2 - (a + b)x + k = 0$, find the value of k.

Solution:

(i) $kx^2 + \sqrt{2} - 4 = 0$, $x = \sqrt{2}$

$x = \sqrt{2}$ is its solution

$$\therefore k\left(\sqrt{2}\right)^2 + \sqrt{2} \times \sqrt{2} - 4 = 0$$

$$\Rightarrow 2k + 2 - 4 = 0$$

$$\Rightarrow 2k - 2 = 0$$

$$\Rightarrow 2k = 2$$

$$\Rightarrow k = \frac{2}{2} = 1$$

$$\therefore k = 1$$

(ii) $x^2 - x(a + b) + k = 0$, $x = a$

$\because x = a$ is its solution

$$\therefore (a)^2 - a(a + b) + k = 0$$

$$\Rightarrow a^2 - a^2 - ab + k = 0 \Rightarrow -ab + k = 0$$

$$\therefore k = ab$$

Question 6.

If 23 and -3 are the roots of the equation $px^2 + 7x + q = 0$, find the values of p and q.

Solution:

$\frac{2}{3}$ and -3 are the roots of the equation $px^2 + 7x + q = 0$

Substituting the value of $x = \frac{2}{3}$ and -3 respectively, we get

$$p\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + q = 0$$

$$\Rightarrow \frac{4}{9}p + \frac{14}{3} + q = 0$$

$$\Rightarrow 4p + 42 + 9q = 0$$

$$\Rightarrow 4p + 9q = -42 \quad \ldots(i)$$

and $p(-3)^2 + 7(-3) + q = 0$

$9p - 21 + q = 0$

$$\Rightarrow 9p + q = 21 \quad \ldots(ii)$$

$q = 21 - 9p$

Substituting the value of q in (i)

$4p + 9(21 - 9p) = -42$

$4p + 189 - 81p = -42$

$-77p = -42 - 189 = -231$

$$p = \frac{-231}{-77} = 3$$

$\therefore q = 21 - 9 \times 3$

$= 21 - 27 = -6$

$\therefore p = 3, q = -6$