

Chapter 8 Algebraic Expressions Ex 8.2

Question 1.

Add:

(i) $7x, -3x$

(ii) $6x, -11x$

(iii) $5x^2, -9x^2$

(iv) $3ab^2, -5ab^2$

(v) $12pq, -13pq$

(vi) $5x^3y, -23x^3y$

Solution:

$$(i) 7x + (-3x) = 7x - 3x = 4x$$

$$(ii) 6x + (-11x) = 6x - 11x = -5x$$

$$(iii) 5x^2 + (-9x^2) = 5x^2 - 9x^2 = -4x^2$$

$$(iv) 3ab^2 + (-5ab^2) = 3ab^2 - 5ab^2 = -2ab^2$$

$$(v) \frac{1}{2}pq + \left(-\frac{1}{3}pq\right) = \frac{1}{2}pq - \frac{1}{3}pq$$

$$= \frac{3pq - 2pq}{6} = \frac{pq}{6} = \frac{1}{6}pq$$

$$(vi) 5x^3y + \left(-\frac{2}{3}x^3y\right) = 5x^3y - \frac{2}{3}x^3y$$

$$= \frac{15x^3y - 2x^3y}{3} = \frac{13x^3y}{3}$$

$$= \frac{13}{3}x^3y$$

Question 2.

Add:

(i) $3x, -5x, 7x$

(ii) $7xy, 2xy, -8xy$

(iii) $-2abc, 3abc, abc$

(iv) $3mn, -5mn, 8mn, -4mn$

(v) $2x^3, 3x^3, -4x^3, -5x^3$

Solution:

$$\begin{aligned}
 & \text{(i) } 3x, -5x, 7x \\
 & = 3x - 5x + 7x \\
 & = (3 - 5 + 7)x \\
 & = (10 - 5)x \\
 & = 5x
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii) } 7xy, 2xy, -8xy \\
 & = 7xy + 2xy - 8xy \\
 & = (7 + 2 - 8)xy \\
 & = (9 - 8)xy \\
 & = xy
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iii) } -2abc, 3abc, abc \\
 & = -2abc + 3abc + abc \\
 & = (-2 + 3 + 1) abc \\
 & = (4 - 2) abc \\
 & = 2abc
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iv) } 3mn, -5mn, 8mn, -4mn \\
 & = 3mn - 5mn + 8mn - 4mn \\
 & = (3 - 5 + 8 - 4) mn \\
 & = (11 - 9) mn \\
 & = 2mn
 \end{aligned}$$

$$\begin{aligned}
 & \text{(v) } 2x^3, 3x^3, -4x^3, -5x^3 \\
 & = 2x^3 + 3x^3 - 4x^3 - 5x^3 \\
 & = (2 + 3 - 4 - 5) x^3 \\
 & = (5 - 9) x^3 \\
 & = -4x^3
 \end{aligned}$$

Question 3.

Simplify the following combining like terms:

(i) $21b - 32 + 7b - 20b$

(ii) $12m^2 - 9m + 5m - 4m^2 - 7m + 10$

(iii) $-z^2 + 13z^2 - 5z + 7z^2 - 15z$

(iv) $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$

(v) $p - (p - q) - (q - p) - q$

(vi) $3a - 2b - ab - (a - b + ab) + 3ab + b - a$

(vii) $(3y^2 + 5y - 4) - (8y - y^2 - 4)$

Solution:

$$(i) 21b - 32 + 7b - 20b$$

$$= 21b + 7b - 20b - 32$$

$$= (21 + 7 - 20)b - 32$$

$$= (28 - 20)b - 32$$

$$= 8b - 32$$

$$(ii) 12m^2 - 9m + 5m - 4m^2 - 7m + 10$$

$$= 12m^2 - 4m^2 - 9m + 5m - 7m + 10$$

$$= (12 - 4)m^2 - (9 - 5 + 7)m + 10$$

$$= 8m^2 - 11m + 10$$

$$(iii) -z^2 + 13z^2 - 5z + 7z^3 - 15z$$

$$= 7z^3 - z^2 + 13z^2 - 5z - 15z$$

$$= 7z^3 + 12z^2 - 20z$$

$$(iv) 5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$$

$$= 5x^2y + 3x^2y + 8xy^2 - 5x^2 + x^2 - 3y^2 - y^2 - 3y^2$$

$$= (5 + 3)x^2y - (5 - 1)x^2 - (3 + 1 + 3)y^2 + 8xy^2$$

$$= 8x^2y - 4x^2 - 7y^2 + 8xy^2$$

$$= 8x^2y + 8xy^2 - 4x^2 - 7y^2$$

$$(v) p - (p - q) - (q - p) - q$$

$$= p - p + q - q + p - q$$

$$= p - p + p + q - q - q$$

$$= p - q$$

$$(vi) 3a - 2b - ab - (a - b + ab) + 3ab + b - a$$

$$= 3a - 2b - ab - a + b - ab + 3ab + b - a$$

$$= 3a - a - a - 2b + b + b - ab + 3ab$$

$$= 3a - 2a - 2b + 2b - ab + 3ab$$

$$= a + ab$$

$$(vii) (3y^2 + 5y - 4) - (8y - y^2 - 4)$$

$$= 3y^2 + 5y - 4 - 8y + y^2 + 4$$

$$= 3y^2 + y^2 + 5y - 8y + 4 - 4$$

$$= 4y^2 - 3y$$

Question 4.

Find the sum of the following algebraic expressions:

(i) $5xy, -7xy, 3x^2$

(ii) $4x^2y, -3xy^2, -5xy^2, 5x^2y$

(iii) $-7mn + 5, 12mn + 2, 8mn - 8, -2mn - 3$

(iv) $a + b - 3, b - a + 3, a - b + 3$

(v) $14x + 10y - 12xy - 13, 18 - 7x - 10y + 8xy, 4xy$

(vi) $5m - 7n, 3n - 4m + 2, 2m - 3mn - 5$

(vii) $3x^3 - 5x^2 + 2x + 1, 3x - 2x^2 - x^3, 2x^2 - 7x + 9$

(viii) $7a^2 - 5a + 2, 3a^2 - 7, 2a + 9, 1 + 2a - 5a^2$

Solution:

$$(i) 5xy, -7xy, 3x^2$$

$$= 5xy - 7xy + 3x^2$$

$$= 3x^2 - 2xy$$

$$(ii) 4x^2y, -3xy^2, -5xy^2, 5x^2y$$

$$= 4x^2y + 5x^2y - 3xy^2 - 5xy^2$$

$$= 9x^2y - 8xy^2$$

$$(iii) -7mn + 5, 12mn + 2, 8mn - 8, -2mn - 3$$

$$\begin{array}{r} -7mn + 5 \\ 12mn + 2 \\ 8mn - 8 \\ -2mn - 3 \\ \hline 11mn - 4 \end{array}$$

$$11mn - 4$$

$$(iv) a + b - 3, b - a + 3, a - b + 3$$

$$\begin{array}{r} a + b - 3 \\ -a + b + 3 \\ a - b + 3 \\ \hline a + b + 3 \end{array}$$

$$a + b + 3$$

(v) $14x + 10y - 12xy - 13$, $18 - 7x - 10y + 8xy$, $4xy$

$$\begin{array}{r} 14x + 10y - 12xy - 13 \\ -7x - 10y + 8xy + 18 \\ + 4xy \\ \hline 7x \qquad \qquad + 5 \end{array}$$

$7x + 5$

(vi) $5m - 7n$, $3n - 4m + 2$, $2m - 3mn - 5$

$$\begin{array}{r} 5m - 7n \\ -4m + 3n + 2 \\ 2m \qquad - 5 - 3mn \\ \hline 3m - 4n - 3 - 3mn \end{array}$$

$3m - 4n - 3mn - 3$

(vii) $3x^3 - 5x^2 + 2x + 1$, $3x - 2x^2 - x^3$, $2x^2 - 7x + 9$

$$\begin{array}{r} 3x^3 - 5x^2 + 2x + 1 \\ -x^3 - 2x^2 + 3x \\ 2x^2 - 7x + 9 \\ \hline 2x^3 - 5x^2 - 2x + 10 \end{array}$$

$2x^3 - 5x^2 - 2x + 10$

(viii) $7a^2 - 5a + 2$, $3a^2 - 7$, $2a + 9$, $1 + 2a - 5a^2$

$$\begin{array}{r} 7a^2 - 5a + 2 \\ 3a^2 \qquad - 7 \\ 2a + 9 \\ -5a^2 + 2a + 1 \\ \hline 5a^2 - a + 5 \end{array}$$

Question 5.

Simplify the following:

(i) $2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2$

(ii) $3xy^2 - 5x^2y + 7xy - 8xy^2 - 4xy + 6x^2y$

(iii) $5x^4 - 7x^2 + 8x - 1 + 3x^3 - 9x^2 + 7 - 3x^4 + 11x - 2 + 8x^2$

Solution:

$$\begin{aligned} \text{(i)} \quad & 2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2 \\ &= 2x^2 + 5x^2 - 3x^2 + 3y^2 - y^2 - 5xy + 6xy \\ &= 4x^2 + 2y^2 + xy \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & 3xy^2 - 5x^2y + 7xy - 8xy^2 - 4xy + 6x^2y \\ &= 3xy^2 - 8xy^2 - 5x^2y + 6x^2y + 7xy - 4xy \\ &= -5xy^2 + x^2y + 3xy \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & 5x^4 - 7x^2 + 8x - 1 + 3x^3 - 9x^2 + 7 - 3x^4 + 11x - 2 + 8x^2 \\ &= 5x^4 - 3x^4 + 3x^3 - 7x^2 - 9x^2 + 8x^2 + 8x + 11x - 1 + 7 - 2 \\ &= 2x^4 + 3x^3 - 8x^2 + 19x + 4 \end{aligned}$$

Question 6.

Subtract:

(i) $-5y^2$ from y^2

(ii) $-7xy$ from $-2xy$

(iii) $a(b - 5)$ from $b(5 - a)$

(iv) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$

(v) $5a^2 - 7ab + 5b^2$ from $3ab - 2b - 2b^2$

(vi) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

(vii) $7xy + 5x^2 - 7y^2 + 3$ from $7x^2 - 8xy + 3y^2 - 5$

(viii) $2x^4 - 7x^2 + 5x + 3$ from $x^4 - 3x^3 - 2x^2 + 3$

Solution:

$$-5y^2 \text{ from } y^2$$

$$= y^2 - (-5y^2)$$

$$= y^2 + 5y^2$$

$$= 6y^2$$

$$(ii) -7xy \text{ from } -2xy$$

$$= -2xy - (-7xy)$$

$$= -2xy + 7xy$$

$$= 5xy$$

$$(iii) a(b - 5) \text{ from } b(5 - a)$$

$$= b(5 - a) - a(b - 5)$$

$$= 5b - ab - ab + 5a$$

$$= 5a + 5b - 2ab$$

$$(iv) -m^2 + 5mn \text{ from } 4m^2 - 3mn + 8$$

$$= 4m^2 - 3mn + 8 - (-m^2 + 5mn)$$

$$= 4m^2 - 3mn + 8 + m^2 - 5mn$$

$$= 5m^2 - 8mn + 8$$

$$\begin{aligned}
 & \text{(v) } 5a^2 - 7ab + 5b^2 \text{ from } 3ab - 2a^2 - 2b^2 \\
 & = (3ab - 2a^2 - 2b^2) - (5a^2 - 7ab + 5b^2) \\
 & = 3ab - 2a^2 - 2b^2 - 5a^2 + 7ab - 5b^2 \\
 & = -7a^2 - 7b^2 + 10ab \\
 & = 10ab - 7a^2 - 7b^2
 \end{aligned}$$

$$\begin{aligned}
 & \text{(vi) } 4pq, -5q^2 - 3p^2 \text{ from } 5p^2 + 3q^2 - pq \\
 & = (5p^2 + 3q^2 - pq) - (4pq - 5q^2 - 3p^2) \\
 & = 5p^2 + 3q^2 - pq - 4pq + 5q^2 + 3p^2 \\
 & = 5p^2 + 3p^2 + 3q^2 + 5q^2 - pq - 4pq \\
 & = 8p^2 + 8q^2 - 5pq
 \end{aligned}$$

$$\begin{aligned}
 & \text{(vii) } 7xy + 5x^2 - 7y^2 + 3 \text{ from } 7x^2 - 8xy + 3y^2 - 5 \\
 & = (7x^2 + 3y^2 - 8xy - 5) - (7xy + 5x^2 - 7y^2 + 3) \\
 & = 7x^2 + 3y^2 - 8xy - 5 - 7xy - 5x^2 + 7y^2 - 3 \\
 & = 7x^2 - 5x^2 + 3y^2 + 7y^2 - 8xy - 7xy - 5 - 3 \\
 & = 2x^2 + 10y^2 - 15xy - 8
 \end{aligned}$$

$$\begin{aligned}
 & \text{(viii) } 2x^4 - 7x^2 + 5x + 3 \text{ from } x^4 - 3x^3 - 2x^2 + 3 \\
 & = (x^4 - 3x^3 - 2x^2 + 3) - (2x^4 - 7x^2 + 5x + 3) \\
 & = x^4 - 3x^3 - 2x^2 + 3 - 2x^4 + 7x^2 - 5x - 3 \\
 & = x^4 - 2x^4 - 3x^3 - 2x^2 + 7x^2 - 5x + 3 - 3 \\
 & = -x^4 - 3x^3 + 5x^2 - 5x
 \end{aligned}$$

Question 7.

Subtract $p - 2q + r$ from the sum of $10p - r$ and $5p + 2q$.

Solution:

Subtract $p - 2q + r$ from the sum of $10p - r$ and $5p + 2q$

By adding $10p - r + 5p + 2q$ and $5p + 2q$, we get

$$= 10p - r + 5p + 2q$$

$$= 15p + 2q - r$$

Now, $(15p + 2q - r) - (p - 2q + r)$

$$= 15p + 2q - r - p + 2q - r$$

$$= 14p + 4q - 2r$$

Question 8.

From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$.

Solution:

$$\text{Sum of } (4 + 3x) + (5 - 4x + 2x^2)$$

$$= 4 + 3x + 5 - 4x + 2x^2$$

$$= 2x^2 - x + 9$$

$$\text{and sum of } 3x^2 - 5x - x^2 + 2x + 5$$

$$= 2x^2 - 3x + 5$$

$$\text{Now, } (2x^2 - x + 9) - (2x^2 - 3x + 5)$$

$$= 2x^2 - x + 9 - 2x^2 + 3x - 5$$

$$= 2x + 4$$

Question 9.

What should be added to $x^2 - y^2 + 2xy$ to obtain $x^2 + y^2 + 5xy$?

Solution:

Let the term added = Z term

$$\text{i.e., } Z \text{ term} + x^2 - y^2 + 2xy = x^2 + y^2 + 5xy$$

$$Z \text{ term} = (x^2 + y^2 + 5xy) - (x^2 - y^2 + 2xy)$$

$$= x^2 + y^2 + 5xy - x^2 + y^2 - 2xy$$

$$= 2y^2 + 3xy$$

The required term is $2y^2 + 3xy$

Question 10.

What should be subtracted from $-7mn + 2m^2 + 3n^2$ to get $m^2 + 2mn + n^2$?

Solution:

Let the term subtracted = Z term

$$-7mn + 2m^2 + 3n^2 - Z \text{ term} = m^2 + 2mn + n^2$$

$$Z \text{ term} = (-7mn + 2m^2 + 3n^2) - (m^2 + 2mn + n^2)$$

$$= (-7mn + 2m^2 + 3n^2) - (m^2 + 2mn + n^2)$$

$$= -7mn + 2m^2 + 3n^2 - m^2 - 2mn - n^2$$

$$= m^2 + 2n^2 - 9mn$$

The required term is $m^2 + 2n^2 - 9mn$

Question 11.

How much is $y^4 - 12y^2 + y + 14$ greater than $17y^3 + 34y^2 - 51y + 68$?

Solution:

The required expression

$$\begin{aligned} &= (y^4 - 12y^2 + y + 14) - (17y^3 + 34y^2 - 51y + 68) \\ &= y^4 - 12y^2 + y + 14 - 17y^3 - 34y^2 + 51y - 68 \\ &= y^4 - 17y^3 - 46y^2 + 52y - 54 \end{aligned}$$

Question 12.

How much does $93p^2 - 55p + 4$ exceed $13p^3 - 5p^2 + 17p - 90$?

Solution:

The required expression

$$\begin{aligned} &= (93p^2 - 55p + 4) - (13p^3 - 5p^2 + 17p - 90) \\ &= 93p^2 - 55p + 4 - 13p^3 - 5p^2 - 17p + 90 \\ &= -13p^3 + 98p^2 - 72p + 94 \end{aligned}$$

Question 13.

What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$?

Solution:

The required expression

$$\begin{aligned} &= (3x^2 - 4y^2 + 5xy + 20) - (-x^2 - y^2 + 6xy + 20) \\ &= 3x^2 - 4y^2 + 5xy + 20 + x^2 + y^2 - 6xy - 20 \\ &= 4x^2 - 3y - xy \end{aligned}$$

Question 14.

From the sum of $2y^2 + 3yz$, $-y^2 - yz - z^2$ and $yz + 2z^2$, subtract the sum of $3y^2 - z^2$ and $-y^2 + yz + z^2$.

Solution:

Sum of $2y^2 + 3yz$, $-y^2 - yz - z^2$ and $yz + 2z^2$

$$\begin{aligned} &= 2y^2 + 3yz - y^2 - yz - z^2 + yz + 2z^2 \\ &= y^2 + z^2 + 3yz \end{aligned}$$

and sum = $3y^2 - z^2 + (-y^2 + yz + z^2)$

$$\begin{aligned} &= 3y^2 - z^2 - y^2 + yz + z^2 \\ &= 2y^2 + yz \end{aligned}$$

Now, $(y^2 + z^2 + 3yz) - (2y^2 + yz)$

$$\begin{aligned} &= y^2 + z^2 + 3yz - 2y^2 - yz \\ &= -y^2 + z^2 + 2yz \\ &= -y^2 + 2yz + z^2 \end{aligned}$$

